

Bibliography and Literature Review on Buckling of Laminated Plates

Osama Mohammed Elmardi Suleiman

*Nile Valley University, Department of Mechanical Engineering
Faculty of Engineering and Technology , Atbara – Sudan
e – mail: osamamm64@gmail.com*

ABSTRACT: The effects of boundary conditions and lamination arrangements (i.e. stacking sequence and orientation of a lamina) were found to be important factors in determining a suitable exact, analytical or semi – analytical method for analyzing buckling loads on laminated plates. It was also found that: as the derivative order of shear deformation increases, the accuracy of stresses, strains, buckling loads ... etc. increases and it doesn't need shear correction factors.

KEYWORDS: Literature review; Buckling; Composites; Laminated plates; Bibliography

1 INTRODUCTION

Composite materials are widely used in a broad spectrum of modern engineering application fields ranging from traditional fields such as automobiles, robotics, day to day appliances, building industry etc... to modern space industries. This is due to their excellent high strength to weight ratio, modulus to weight ratio, and the controllability of the structural properties with the variation of fiber orientation, stacking scheme and the number of laminates. Among the various aspects of the structural performance of structures made of composite materials is the mechanical behaviour of rectangular laminated plates which has drawn much attention. In particular, consideration of the buckling phenomena in such plates is essential for the efficient and reliable design and for the safe use of the structural element. Due to the anisotropic and coupled material behaviour, the analysis of composite laminated plates is generally more complicated than the analysis of homogeneous isotropic ones.

The members and structures composed of laminated composite material are usually very thin, and hence more prone to buckling. Buckling phenomenon is critically dangerous to structural components because the buckling of composite plates usually occurs at a lower applied stress and generates large deformations. This led to a focus on the study of buckling behaviour in composite materials. General introductions to the buckling of elastic structures and of laminated plates can be found in e.g. Refs. [1] and [2]. However, these available Curves

and data are restricted to idealized loading, namely, uniaxial or biaxial uniform compression.

2 THE PAST WORK OF BUCKLING ANALYSIS

Due to the importance of buckling considerations, there is an overwhelming number of investigations available in which corresponding stability problems are considered by a wide variety of analysis methods which may be of a closed – form analytical nature or may be sorted into the class of semi – analytical or purely numerical analysis method.

Closed – form exact solutions for the buckling problem of rectangular composite plates are available only for limited combinations of boundary conditions and laminated schemes. These include cross – ply symmetric and angle – ply anti – symmetric rectangular laminates with at least two opposite edges simply supported, and similar plates with two opposite edges clamped but free to deflect (i.e. guided clamp) or with one edge simply supported and the opposite edge with a guided clamp. Most of the exact solutions discussed in the monographs of Whitney [3] who developed an exact solution for critical buckling of solid rectangular orthotropic plates with all edges simply supported , and of Reddy [4] and Leissa and Kang [5]. Bao et al. [6] developed an exact solution for two edges simply supported and two edges clamped, and Robinson [7] who developed an exact solution for the critical buckling stress of an orthotropic sandwich plate with all edges simply supported.

For all other configurations, for which only approximated results are available, several semi – analytical and numerical techniques have been developed. The Rayleigh – Ritz method [8], the finite strip method (FSM) [9], the element free Galerkin method (EFG) [10], the differential quadrature technique [11], the moving least square differential quadrature method [12], and the most extensively used finite element method (FEM) [13] are the most common ones. The Kantorovich method (KM) [14], which is a different and in most cases advantageous semi – analytical method, combines a variation approach of closed – form solutions and an iterative procedure. The method assumes a solution in the form of a sum of products of functions in one direction and functions in the other direction. Then, by assuming the function in one direction, the variation problem of the plate reduces to a set of ordinary differential equations. In the case of buckling analysis, the variation problem reduces to an ordinary differential eigenvalue and eigenfunction problem. The solution of the resulting problem is an approximate one, and its accuracy depends on the assumed functions in the first direction. The extended Kantorovich method (EKM), which was proposed by Kerr [15], is the starting point for an iterative procedure, where the solution obtained in one direction is used as the assumed functions in the second direction. After repeating this process several times, convergence is obtained. The single term extended

Kantorovich method was employed for a buckling analysis of rectangular plates by several researches. Eienberger and Alexandrov [16] used the method for the buckling analysis of isotropic plates with variable thickness. Shufrin and Eisenberger [17] extended the solution to thick plates with constant and variable thickness using the first and higher order shear deformation theories. Ungbhakorn and Singhatanadgid [18] extended the solution to buckling of symmetrically cross – ply laminated rectangular plates. The multi – term formulation of the extended Kantorovich approach to the simplest samples of rectangular isotropic plates was presented by Yuan and Jin [19]. This study showed that the additional terms in the expansion can be used in order to improve the solution.

March and Smith [20] found an approximate solution for all edges clamped. Also, Chang et al. [21] developed approximate solution to the buckling of rectangular orthotropic sandwich plate with two edges simply supported and two edges clamped or all edges clamped using the March – Erickson method and an energy technique. Jiang et al. [22] developed solutions for local buckling of rectangular orthotropic hat – stiffened plates with edges parallel to the stiffeners were simply supported or clamped and edges parallel to the stiffeners were free, and Smith [23] presented solutions bounding the local buckling of hat stiffened plates by considering the section between stiffeners as simply supported or clamped plates.

Many authors have used finite element method to predict accurate in – plane stress distribution which is then used to solve the buckling problem. Cook [24] has clearly presented an approach for finding the buckling strength of plates by first solving the linear elastic problem for a reference load and then the eigenvalue problem for the smallest eigenvalue which then multiplied by the reference load gives the critical buckling load of the structure. An excellent review of the development of plate finite elements during the past 35 years was presented by Yang et al. [25].

Many buckling analysis of composite plates available in the literature are usually realized parallel with the vibration analyses, and are based on two – dimensional plate theories which may be classified as classical and shear deformable ones. Classical plate theories (CPT) do not take into account the shear deformation effects and over predict the critical buckling loads for thicker composite plates, and even for thin ones with a higher anisotropy. Most of the shear deformable plate theories are usually based on a displacement field assumption with five unknown displacement components. As three of these components corresponded to the ones in CPT, the additional ones are multiplied by a certain function of thickness coordinate and added to the displacements field of CPT in order to take into account the shear deformation effects. Taking these functions as linear and cubic forms leads to the so – called uniform or Mindlin shear deformable plate theory (USDPT) [26], and parabolic shear deformable plate theories (PSDPT) [27] respectively. Different forms were also

employed such as hyperbolic shear deformable plate theory (HSDPT) [28], and trigonometric or sine functions shear deformable plate theory (TSDPT) [29] by researchers. Since these types of shear deformation theories do not satisfy the continuity conditions among many layers of the composite structures, the zig – zag type of the plate theories introduced by Di Sciuva [30], and Cho and Parmeter [31] in order to consider interlaminar stress continuities. Recently, Karama et al. [32] proposed a new exponential function (i.e. exponential shear deformable plate theory (ESDPT)) in the displacement field of the composite laminated structures for the representation of the shear stress distribution along the thickness of the composite structures and compared their result for static and dynamic problem of the composite beams with the sine model.

Within the classical lamination theory, Jones [33] presented a closed – form solution for the buckling problem of cross – ply laminated plates with simply supported boundary conditions. In the case of multi – layered plates subjected to various boundary conditions which are different from simply supported boundary conditions at all of their four edges, the governing equations of the buckling of the composite plates do not admit an exact solution, except for some special arrangements of laminated plates. Thus, for the solution of these types of problems, different analytical and / or numerical methods are employed by various researchers. Baharlou and Leissa [8] used the Ritz method with simple polynomials as displacement functions, within the classical theory, for the problem of buckling of cross and angle – ply laminated plates with arbitrary boundary conditions and different in – plane loads. Narita and Leissa [34] also applied the Ritz method with the displacement components assumed as the double series of trigonometric functions for the buckling problem of generally symmetric laminated composite rectangular plates with simply supported boundary conditions at all their edges. They investigated the critical buckling loads for five different types of loading conditions which are uniaxial compression (UA – C), biaxial compression (BA – C), biaxial compression – tension (BA – CT), and positive and negative shear loadings.

The higher – order shear deformation theories can yield more accurate inter – laminate stress distributions. The introduction of cubic variation of displacement also avoids the need for shear correction displacement. To achieve a reliable analysis and safe design, the proposals and developments of models using higher order shear deformation theories have been considered. Lo et al. [35] reviewed the pioneering work on the field and formulated a theory which accounts for the effects of transverse shear deformation, transverse strain and non – linear distribution of the in – plane displacements with respect to the thickness coordinate. Third – order theories have been proposed by Reddy [36], Librescu [37], Schmidt [38], Murty [39], Levinson [40], Seide [41], Murthy [42], Bhimaraddi [43], Mallikarjuna and Kant [44], Kant and Pandya [45] among others. Pioneering work and overviews in the field covering closed – form solutions and finite element models can be found in Reddy [46],

Mallikarjuna and kant [44] , Noor and Burton [47] , Bert [48], Kant and Kommineni [49], and Reddy and Robbins [50] among others.

For the buckling analysis of the cross – ply laminated plates subjected to simply supported boundary conditions at their opposite two edges and different boundary conditions at the remaining ones Khdeir [51] used a parabolic shear deformation theory and applied the state – space technique. Hadian and Nayfeh [52], on the basis of the same theory and for the same type of problem, needed to modify the technique due to ill – conditioning problems encountered especially for thin and moderately thick plates. The buckling analyses of completely simply supported cross – ply laminated plates were presented by Fares and Zenkour [53], who added a non – homogeneity coefficient in the material stiffnesses within various plate theories , and by Matsunaga [54] who employed a global higher order plate theory. Gilat el al. [55] also investigated the same type of problem on the basic of a global – local plate theory where the displacement field is composed of global and local contributions, such that the requirement of the continuity conditions and delaminations effects can be incorporated into formulation.

Many investigations have been reported for static and stability analysis of composite laminates using different traditional methods. Pagano [56] developed an exact three – dimensional (3 – D) elasticity solution for static analysis of rectangular bi – directional composites and sandwich plates. Noor [57] presented a solution for stability of multi – layered composite plates based on 3 – D elasticity theory by solving equations with finite difference method. Also, 3 – D elasticity solutions are presented by Gu and Chattopadhyay [58] for the buckling of simply supported orthotropic composite plates. When the problem is reduced from a three – dimensional one (3 – D) to a two dimensional case to contemplate more efficiently the computational analysis of plate composite structures, the displacement based theories and the corresponding finite element models receive the most attention [59].

Bifurcation buckling of laminated structures has been investigated by many researchers without considering the flatness before buckling [60]. This point was first clarified for laminated composite plates for some boundary conditions and for some lamina configurations by Leissa [60]. Qatu and Leissa [61] applied this result to identify true buckling behaviour of composite plates. Elastic bifurcation of plates have been extensively studied and well documented in standard texts e.g. [62], research monographs [63] and journal papers [64].

3 CONCLUSIONS

Comprehensive bibliography and literature review on buckling of composite laminated plates were presented and discussed thoroughly. Exact, analytical and semi – analytical solutions in buckling of laminates were analyzed using

different factors which include boundary conditions plate dimensions and lamination scheme.

Development of plate theories from classical plate theory through first order shear deformation, and to higher order shear deformation theories were considered in the analysis of buckling. It was found that higher order shear deformation theories can yield more accurate inter – laminate stresses and also avoids the need for shear correction displacement.

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