Charts for Bending Moment Coefficients for Continuous Beams

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Abstract: **This paper presents the results of studies on the bending moments in continuous beams caused by uniformly distributed load exerted over full span lengths of beam. An elastic analysis based on moment distribution method is attempted to deduce the coefficients of negative and positive moments for continuous beams through computing 2000 examples with adopting beams of two spans, three spans and four spans and changing their spans lengths. The results are summarized in several charts for the convenience of practical estimation of these coefficients of bending moments. The values of the coefficients of negative and positive bending moments can be easily derived from the charts with a sufficient accuracy emphasized by some numerical examples carried in this paper.**

I. Introduction:

Structural analysis for structures is the one of the basic means to enter to the design of structures through calculating the design data represented on the forces like bending moment, shear force, ...etc. there are many methods of structural analysis include analytical methods which depend on using manual calculation and also include numerical methods which most of them used computers. In addition to existence of wide spreading of engineering software used in structural analysis. In spite of talking mentioned, there is needing of using simplified and quick methods instead of using more calculations or needing for using software in some practice. To go with this talk most of the design codes adopted this approach. For example ACI Code [1], BS8110 [2], EC2 [3] and more. The codes give the coefficients of bending moments and shear force under some provisions for continuous beams and slabs. As an example the ACI code provisions require two or more continuous spans, spans of equal length or having the larger of two adjacent spans not being greater than the shorter by more than 20 percent, loads being uniformly distributed, unfactored live load not exceeding three times unfactored dead load, and the members being prismatic. Comprehensive work was done by Reynolds and Steedman [4] to calculate bending moments, shear forces and deflections for simple and continuous beams under different conditions of loading and supports and that by using Equations and/or Charts and Tables. A shear and moment formulas with diagrams for simply supported beams and continuous beams of two spans were done by American Wood Council [5].Khuda and Anwar [6] were studying the behavior of beams and developed moment coefficients for beams of different spans and varying span ratio. Fathelrahman et al [7] deduced negative moment coefficients for continuous beams of equal and unequal spans.

This paper generates charts for coefficients of negative and positive bending moments for continuous beams of 2-spans, 3-spans and 4-spans of different spans lengths controlled by spans ratio depending on the fist span and ranging between 0.5 to 2.0 for the 2 and 3-spans and for 4-spans the ratio ranging between 0.5 to 1.6 all these with keeping the ratio for the first span equal 1.0. The study was carried under assumptions that the beams are carried uniformly distributed load full spans and the cross sections are prismatic. The analysis was done according to the moment distribution method. With changing the ratio of spans, a number of 16 examples for 2-spans were obtained, while for 3-spans 256 examples were obtained and for the 4-spans 1728 examples were obtained.

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II. Moment distribution method:

The method was invented by Professor Hardy Cross of University of Illinois of U.S.A. in 1930 [8]. The method can be used to analyze all types of statically indeterminate beams or rigid frames. The moment distribution method involves the following steps:

a. *Determination of* ***fixed end moments*** *due to externally applied loads for all the members.*
b. *Determination of* ***distribution factors*** *for members meeting at each joint.*
c. *The sum of the fixed end moments at each joint is calculated (unbalanced moment). Next calculate the* ***balancing moments*** *which is equal in magnitude but opposite in sign to this sum.*
d. *At each joint distribution of* ***balancing moments*** *to each member in the ratios of* ***distribution factors****.*
e. ***Carry over*** *half the distributed moments to the other end of each member.*
f. *Repeat the cycle (d) and (e) once it is completed for all the joints of the structure.*

This process continues till sufficient accuracy of results is achieved with attention to that the process should stop only after a distribution and never after a *carry over.*

The terms fixed end moments, distribution factors and carry over, are well illustrated in Reference [7].

III. Models of Continuous Beam

The 2000 examples used as model of continuous were analyzed under the following assumptions:

- The cross sections of beams are prismatic.
- All the supports are hinges.
- The loads applied is uniformly distributed load full span and have a unit value (w=1.0kN/m).
- The span lengths are controlled by the spans ratio (S), which all spans were divided by the first span length (L1) this gives S1 = 1.0.

Figures 1,2 and 3 illustrate these assumption.

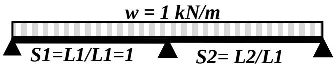


Figure 1: Continuous Beam of 2-spans where S2 ranges between 0.5 and 2.0

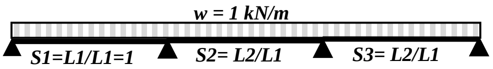


Figure 2: Continuous Beam of 3-spans where S2 and S3 range between 0.5 and 2.0

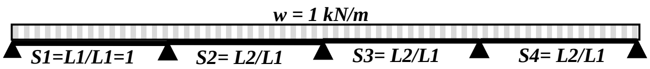


Figure 3: Continuous Beam of 4-spans where S2, S3 and S4 range between 0.5 and 1.6

IV. Calculation of Negative Moment Coefficients:

With following steps of moment distribution method the negative moments at supports have been calculated with aid of using spreadsheets. The coefficients of moments can be calculated by dividing the support moments by the span length. Since the support moment is in between two spans at left and at right, an average of these spans is used to calculate the coefficient of moment. Later the same average is used to calculate the bending moment once the coefficient being derived from chart. Accordingly the coefficient C can be calculated using Equation (1).

$$C = \frac{M_s}{½(L_L + L_R)} \qquad (1)$$

where:

C is the negative moment coefficient at the internal supports (with taken w=1.0).

Ms is the negative moment at support.

L_L is the length if span to left of support

L_R is the span length right to the support.

V. Calculation of Positive Moments Coefficients:

The calculation of positive moment coefficient is depends on the maximum positive moment at spans which is calculated using the values of negative moment by following these steps:

1. *find shear force at ends of each element in the manner described in Figure 4.*

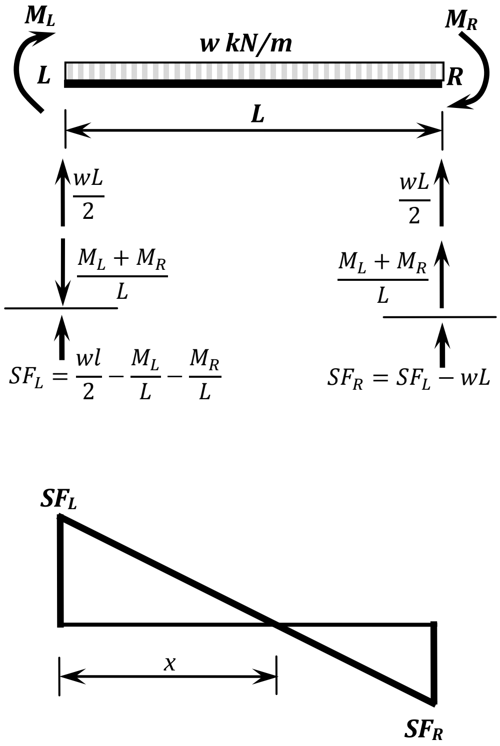


Figure 4:Shear Force Calculation by Balancing the Forces at the End of the Element

Where:

M_L and M_R are the supports moments at left and right of the element respectively.

SF_L and SF_R are shear force at left and right of the element respectively.

w is the uniformly distributed load applied over the element (here is taken 1.0).

L is the element span length .

2. *Find the point of zero shear force to locate the point of maximum positive moment by calculating the distance x from left side using Equation (2):*

$$x = \frac{SF_L}{w} \qquad (2)$$

3. *Calculate the maximum positive moment (M_{pos}) by using Equation (3):*

$$M_{pos} = M_L + SF_L(x) - ½w(x)^2 \qquad (3)$$

4. *Calculate the positive moment coefficient (D) by divide the value of positive moment by the span length, this shown if Equation (4) (this case of w = 1.0):*

$$D = \frac{M_{Pos}}{L} \qquad (4)$$

The negative and positive moments coefficients for the 2, 3 and 4 spans are coded as shown if Figure 5.

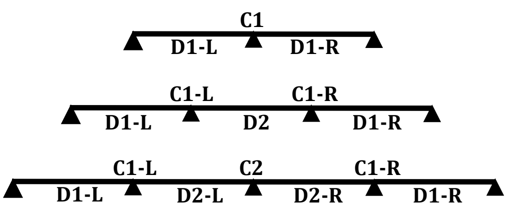


Figure 5: Naming of Positive and Negative Moments Coefficients for 2, 3 an4 spans

Charts for Negative and Positive Moments Coefficients for the continuous beams of 2-Spans :

In the following is the charts to extract the negative moments coefficient (C1) and positive moment coefficients (D1-L and D1-R) for the different ratios of S2 range between 0.5 and 2.0

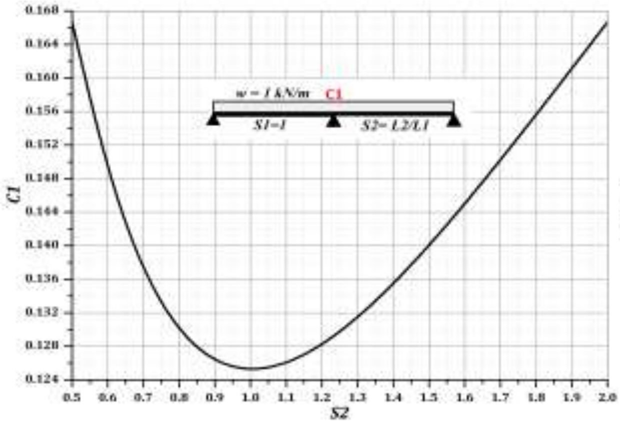


Chart 1: Coefficient C1 for 2-spans, S2 (0.5-2.0)

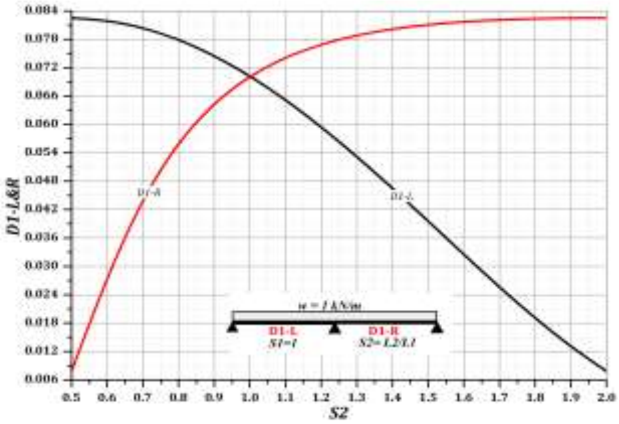


Chart 2: Coefficients D1-L and D1-R for 2-spans, S2 (0.5-2.0)

Charts for Negative Moments Coefficients for the continuous beams of 3-Spans :

In the following is the charts to extract the negative moments coefficient (C1-L and C1-R) for the different ratios of S2 and S3 range between 0.5 and 2.0.

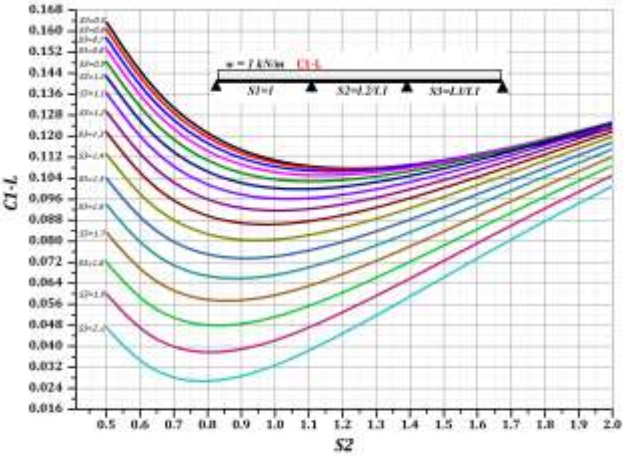


Chart 3: Coefficient C1-L for 3-spans, S2 & S3 (0.5-2.0)

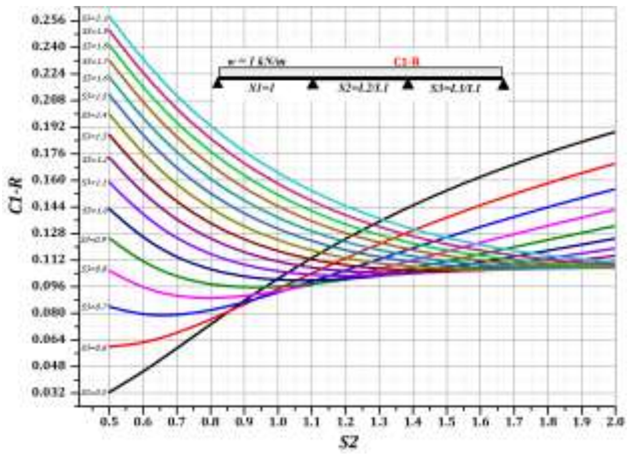


Chart 4: Coefficient C1-R for 3-spans, S2 & S3 (0.5-2.0)

Charts for Positive Moments Coefficients for the continuous beams of 3-Spans :

In the following is the charts to extract the positive moment coefficients (D1-L, D2 and D1-R) for the different ratios of S2 and S3 range between 0.5 and 2.0

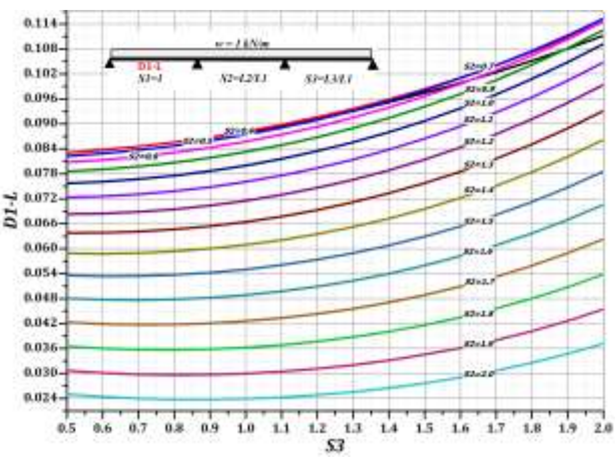


Chart 5: Coefficient D1-L for 3-spans, S2 & S3 (0.5-2.0)

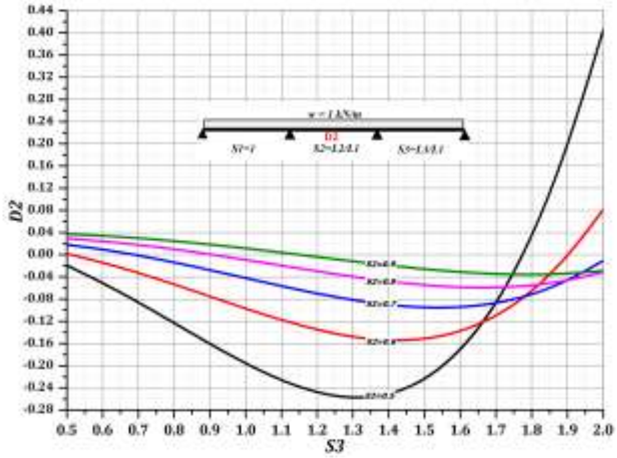


Chart 6: Coefficient D2 for 3-spans, S2 (0.5-0.9), S3 (0.5-2.0)

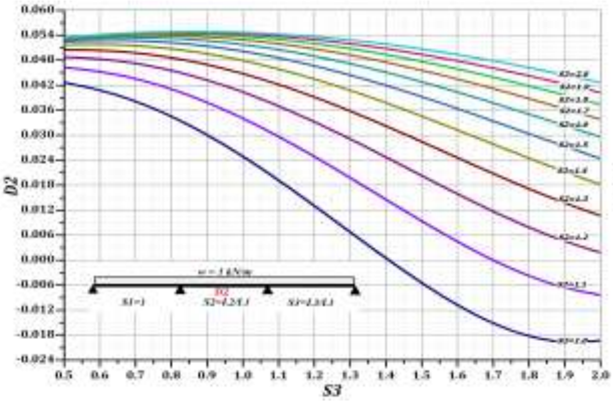


Chart 7: Coefficient D2 for 3-spans, S2 (1.0-2.0), S3 (0.5-2.0)

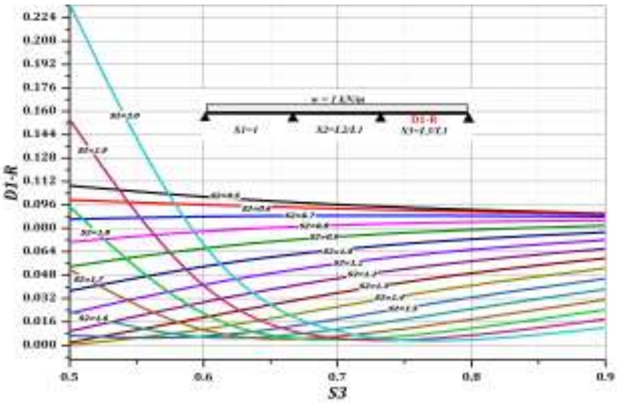


Chart 8: Coefficient D1-R for 3-spans, S2 (0.5-2.0), S3 (0.5-0.9)

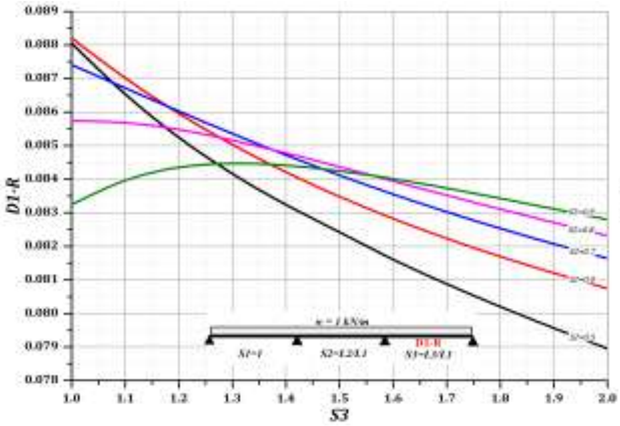


Chart 9: Coefficient D1-R for 3-spans, S2 (0.5-0.9), S3 (1.0-2.0)

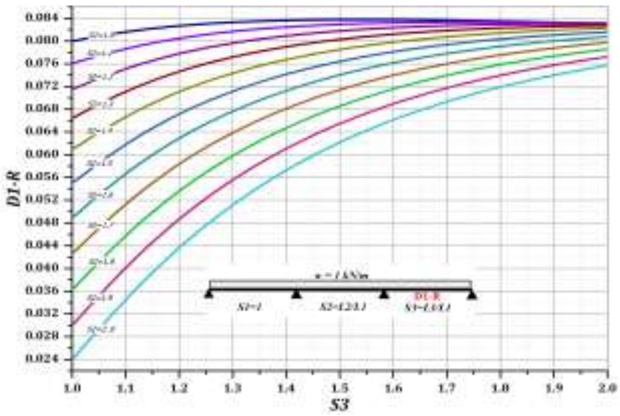


Chart 10: Coefficient D1-R for 3-spans, S2 (1.0-2.0), S3 (1.0-2.0)

Charts for Negative Moments Coefficients for the continuous beams of 4-Spans :

In the following is the charts to extract the negative moments coefficient (C1-L, C2 and C1-R) for the different ratios of S2, S3 and S4 range between 0.5 and 1.6.

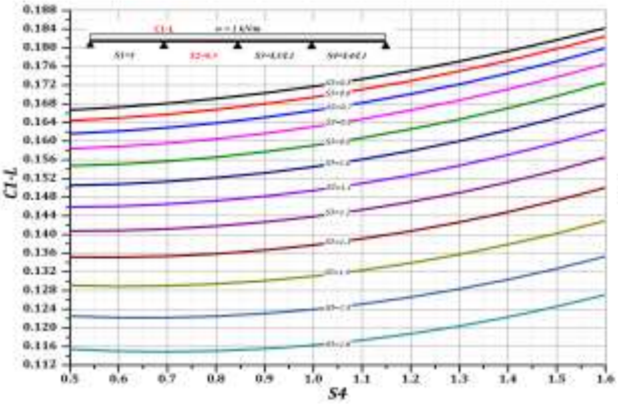


Chart 11: Coefficient C1-L for 4-spans, S2 (0.5), S3 & S4 (0.5-1.6)

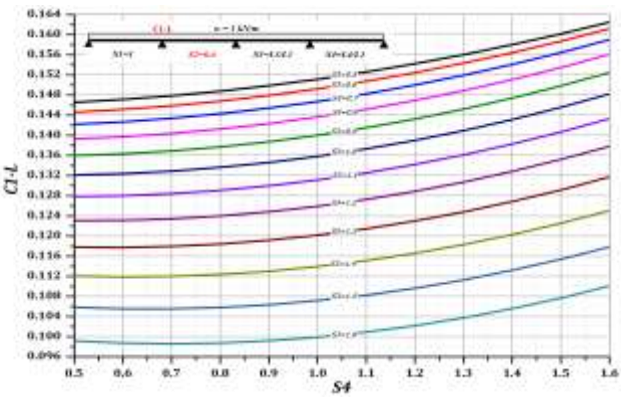


Chart 12: Coefficient C1-L for 4-spans, S2 (0.6), S3 & S4 (0.5-1.6)

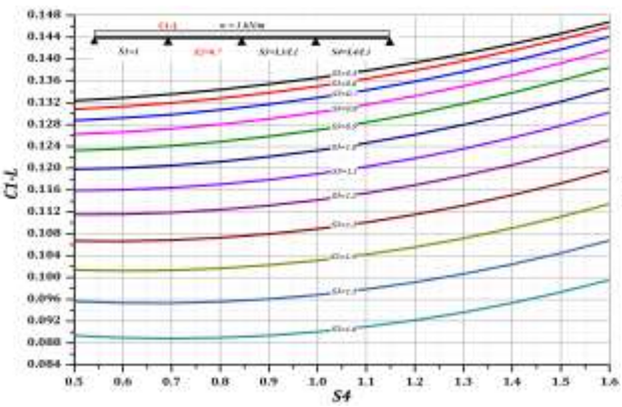


Chart 13: Coefficient C1-L for 4-spans, S2 (0.7), S3 & S4 (0.5-1.6)

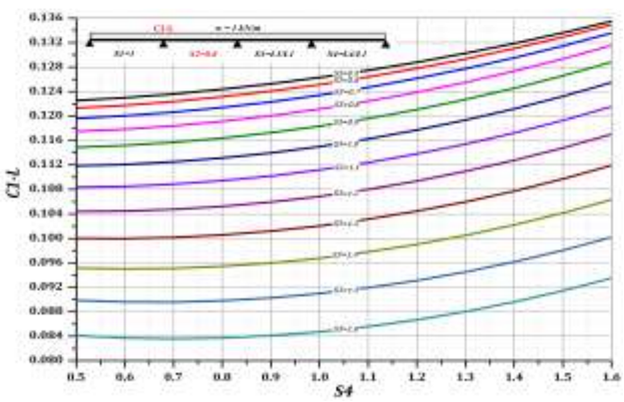


Chart 14: Coefficient C1-L for 4-spans, S2 (0.8), S3 & S4 (0.5-1.6)

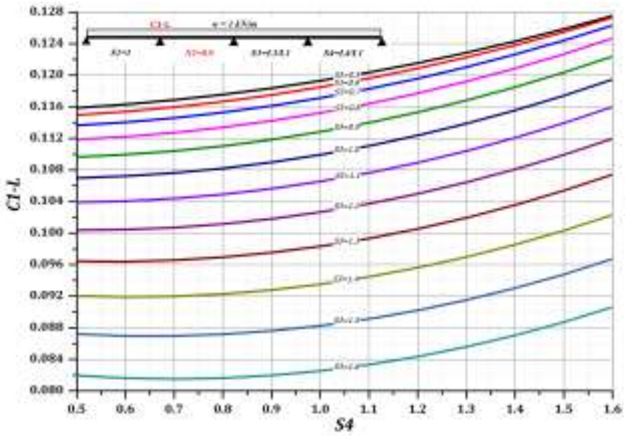


Chart 15: Coefficient C1-L for 4-spans, S2 (0.9), S3 & S4 (0.5-1.6)

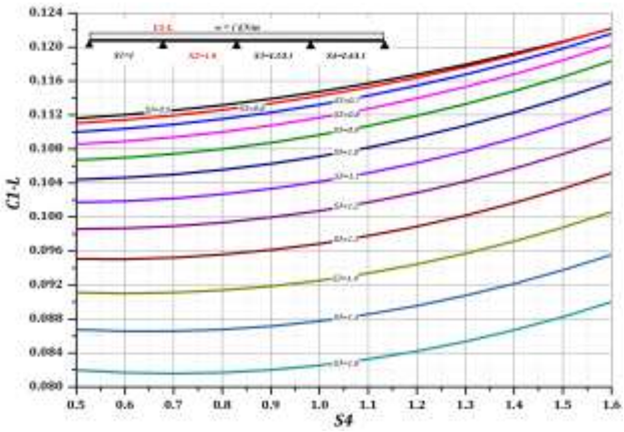


Chart 16: Coefficient C1-L for 4-spans, S2 (1.0), S3 & S4 (0.5-1.6)

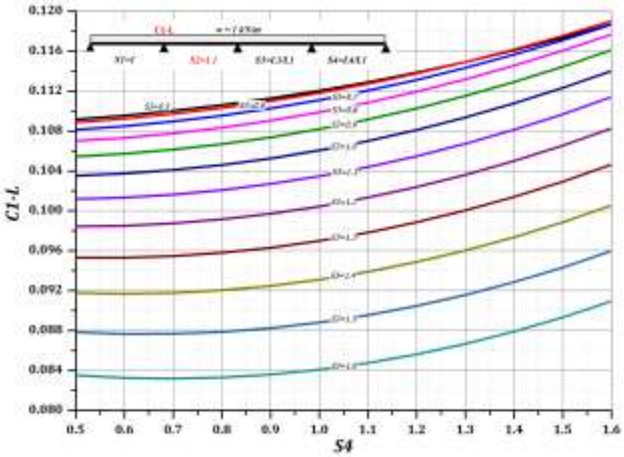


Chart 17: Coefficient C1-L for 4-spans, S2 (1.1), S3 & S4 (0.5-1.6)

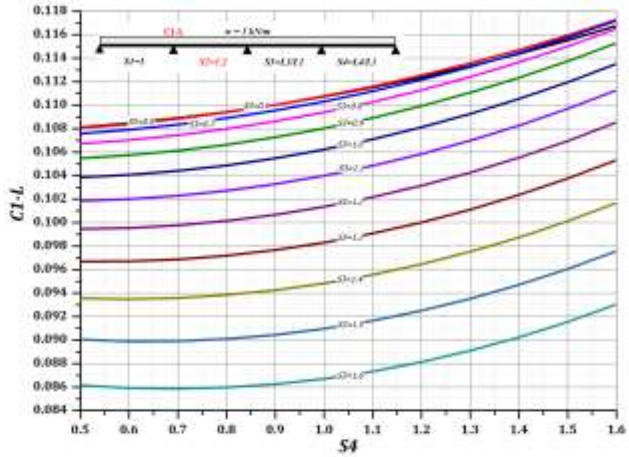


Chart 18: Coefficient C1-L for 4-spans, S2 (1.2), S3 & S4 (0.5-1.6)

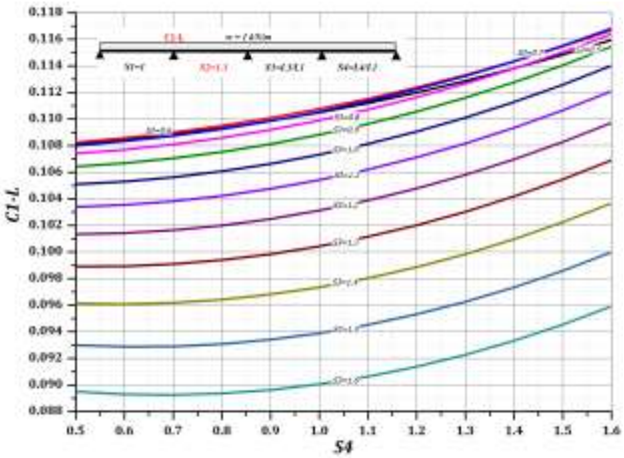


Chart 19: Coefficient C1-L for 4-spans, S2 (1.3), S3 & S4 (0.5-1.6)

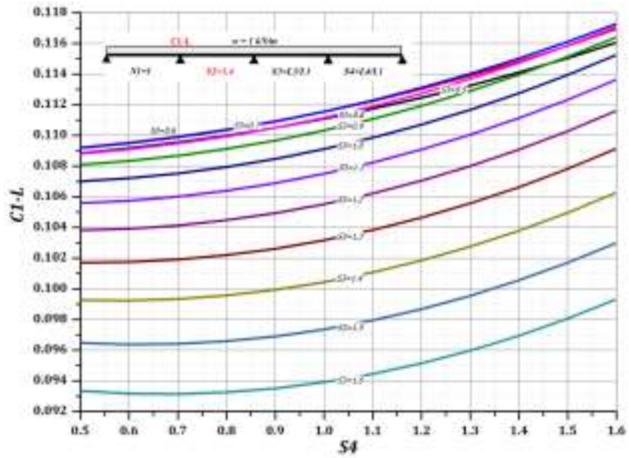


Chart 20: Coefficient C1-L for 4-spans, S2 (1.4), S3 & S4 (0.5-1.6)

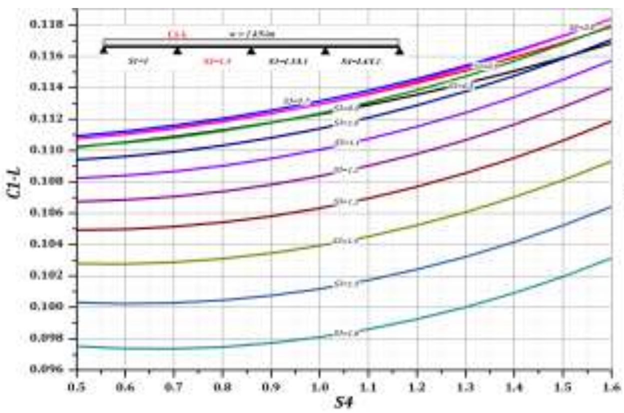


Chart 21: Coefficient C1-L for 4-spans, S2 (1.5), S3 & S4 (0.5-1.6)

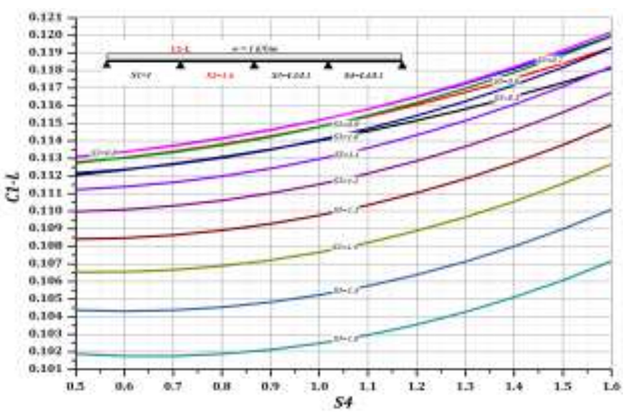


Chart 22: Coefficient C1-L for 4-spans, S2 (1.6), S3 & S4 (0.5-1.6)

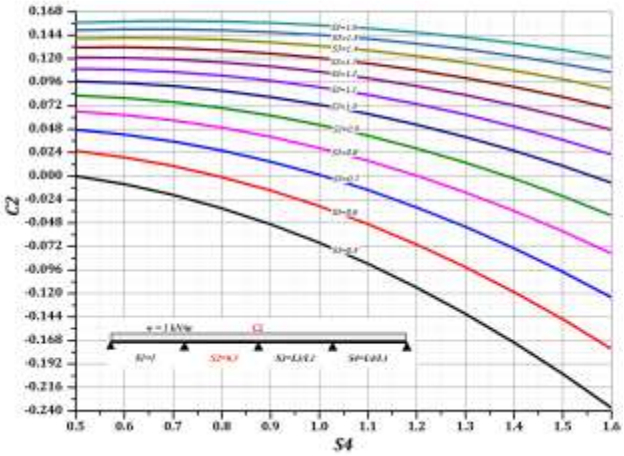


Chart 23: Coefficient C2 for 4-spans, S2 (0.5), S3 & S4 (0.5-1.6)

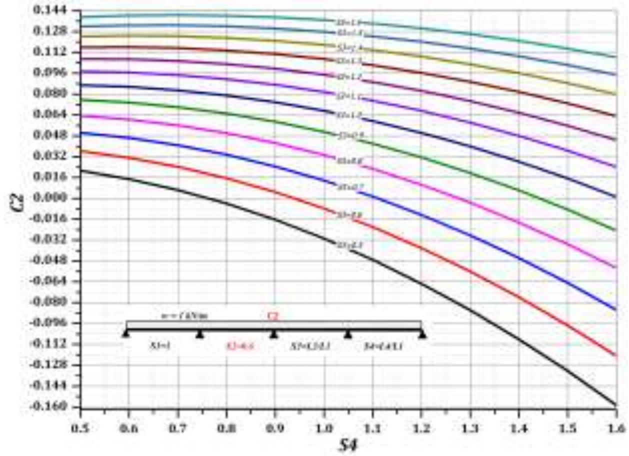


Chart 24: Coefficient C2 for 4-spans, S2 (0.6), S3 & S4 (0.5-1.6)

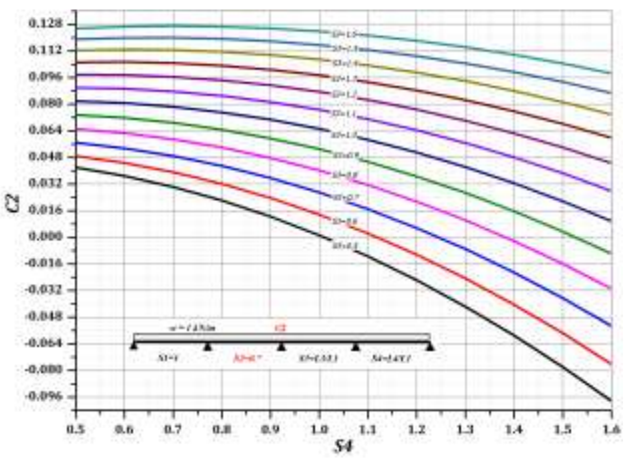


Chart 25: Coefficient C2 for 4-spans, S2 (0.7), S3 & S4 (0.5-1.6)

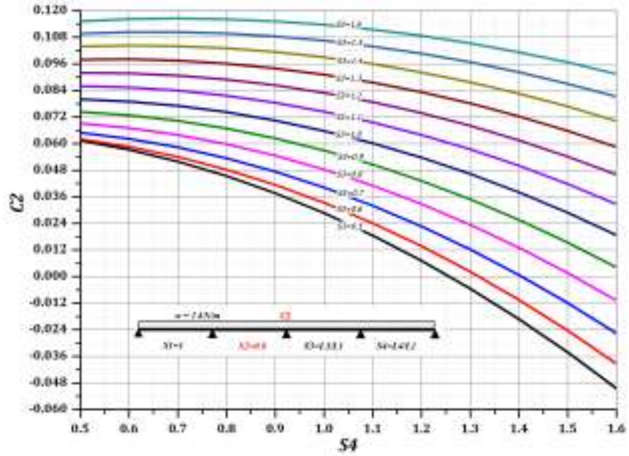


Chart 26: Coefficient C2 for 4-spans, S2 (0.8), S3 & S4 (0.5-1.6)

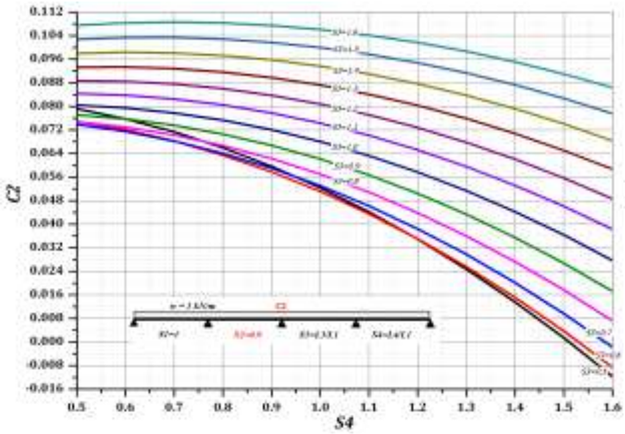


Chart 27: Coefficient C2 for 4-spans, S2 (0.9), S3 & S4 (0.5-1.6)

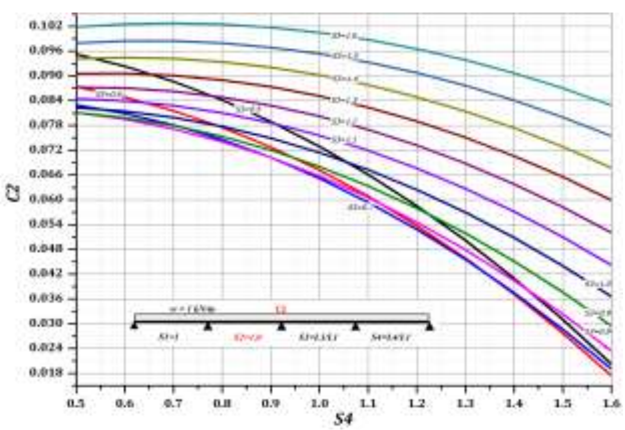
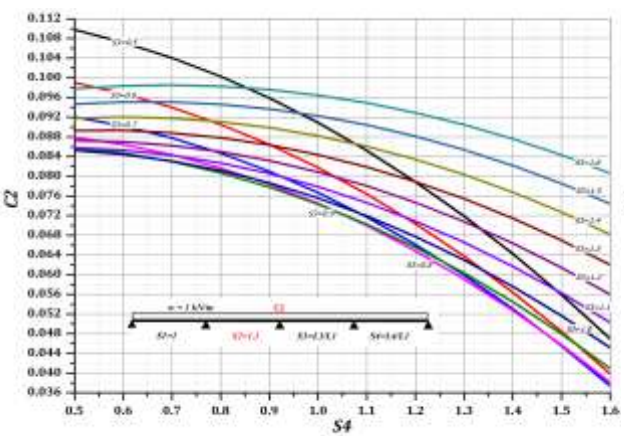


Chart 28: Coefficient C2 for 4-spans, S2 (1.0), S3 & S4 (0.5-1.6)



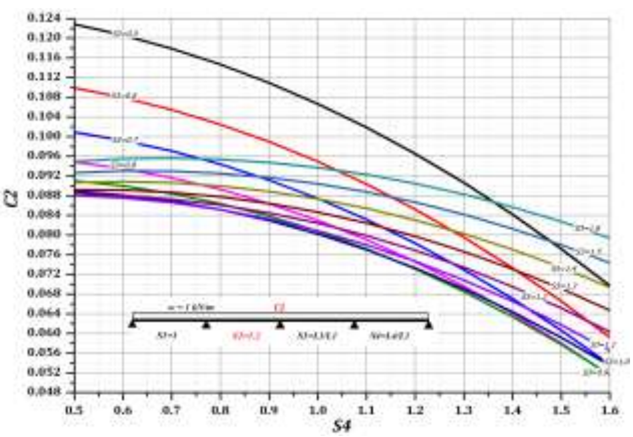
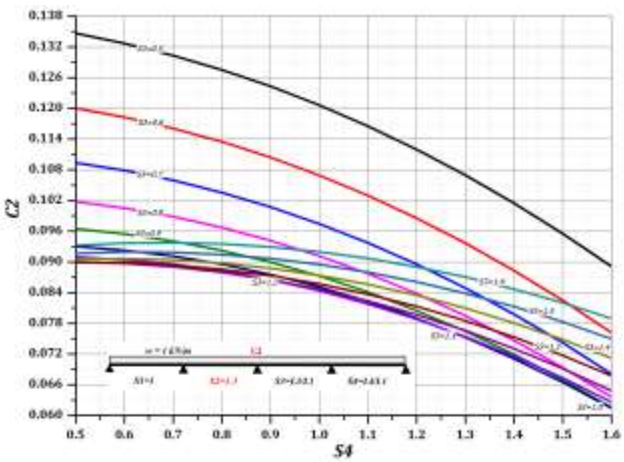


Chart 29: Coefficient C2 for 4-spans, S2 (1.1), S3 & S4 (0.5-1.6) **Chart 30**: Coefficient C2 for 4-spans, S2 (1.2), S3 & S4 (0.5-1.6)



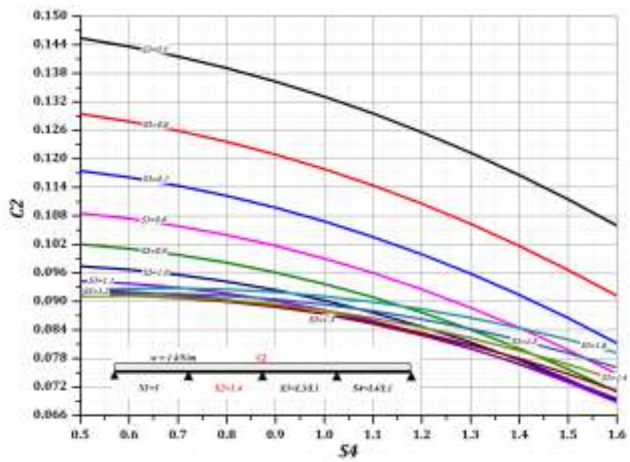
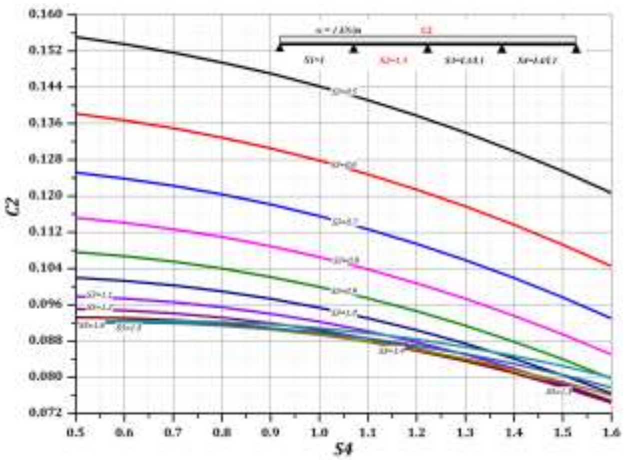


Chart 31: Coefficient C2 for 4-spans, S2 (1.3), S3 & S4 (0.5-1.6) **Chart 32**: Coefficient C2 for 4-spans, S2 (1.4), S3 & S4 (0.5-1.6)



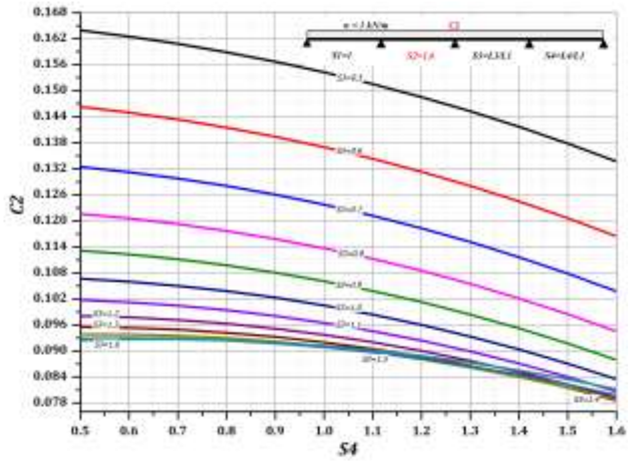
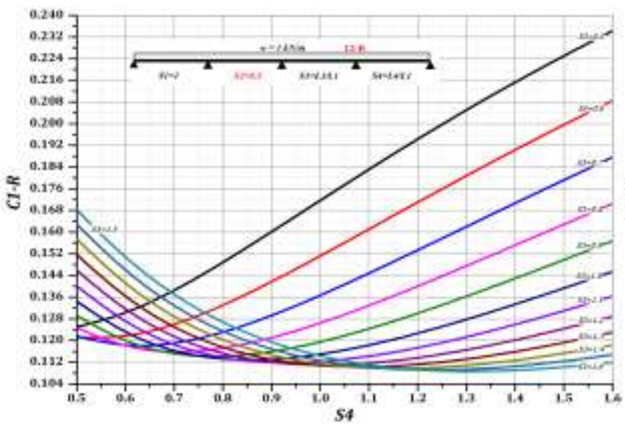


Chart 33: Coefficient C2 for 4-spans, S2 (1.5), S3 & S4 (0.5-1.6) **Chart 34**: Coefficient C2 for 4-spans, S2 (1.6), S3 & S4 (0.5-1.6)



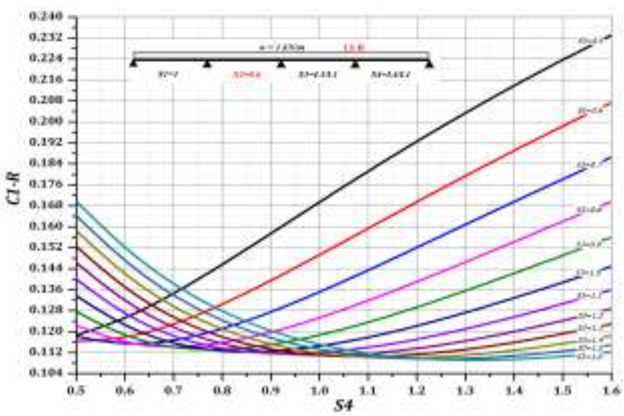


Chart 35: Coefficient C1-R for 4-spans, S2 (0.5), S3 & S4 (0.5-1.6) **Chart 36**: Coefficient C1-R for 4-spans, S2 (0.6), S3 & S4 (0.5-1.6)

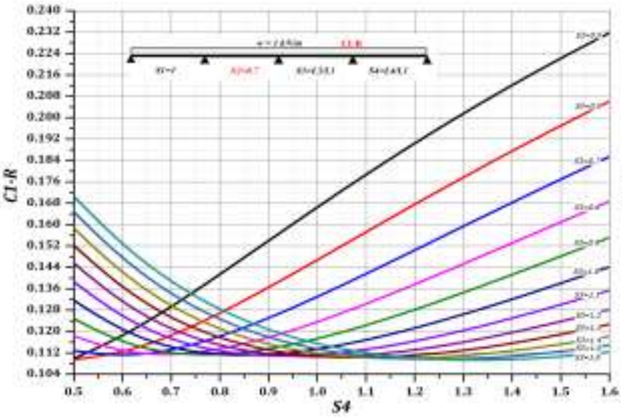


Chart 37: Coefficient C1-R for 4-spans, S2 (0.7), S3 & S4 (0.5-1.6)

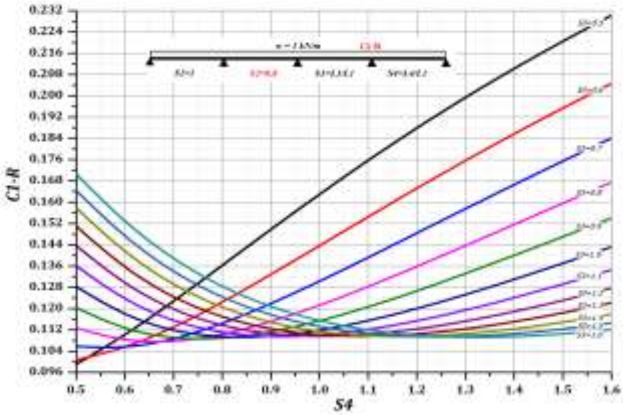


Chart 38: Coefficient C1-R for 4-spans, S2 (0.8), S3 & S4 (0.5-1.6)

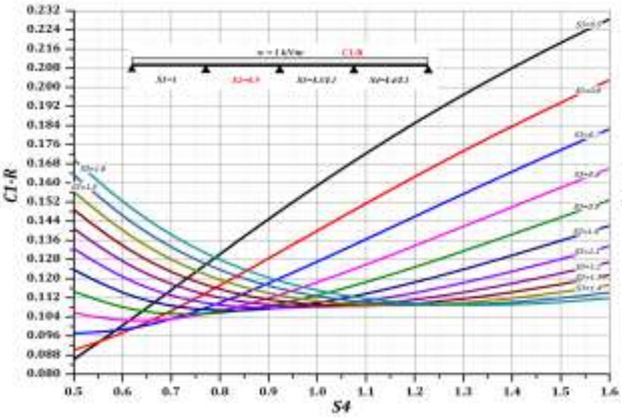


Chart 39: Coefficient C1-R for 4-spans, S2 (0.9), S3 & S4 (0.5-1.6)

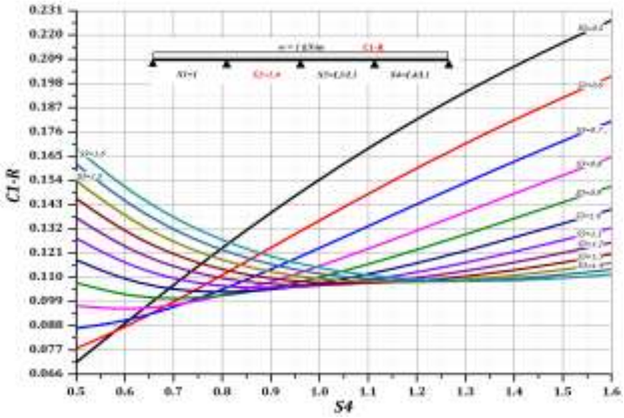


Chart 40: Coefficient C1-R for 4-spans, S2 (1.0), S3 & S4 (0.5-1.6)

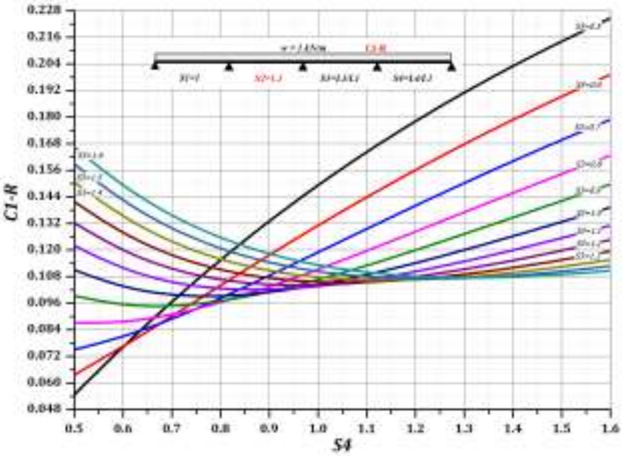


Chart 41: Coefficient C1-R for 4-spans, S2 (1.1), S3 & S4 (0.5-1.6)

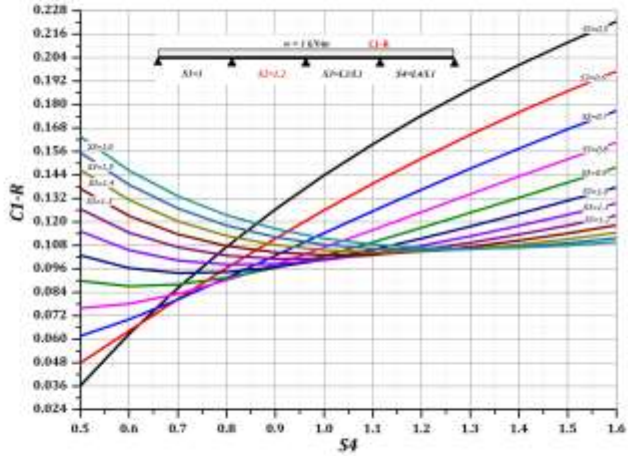


Chart 42: Coefficient C1-R for 4-spans, S2 (1.2), S3 & S4 (0.5-1.6)

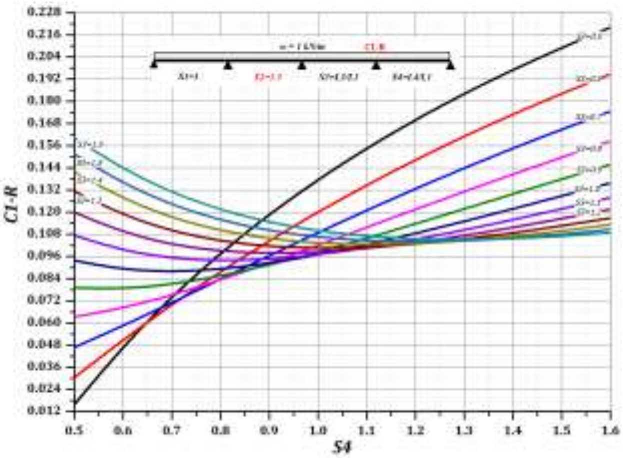


Chart 43: Coefficient C1-R for 4-spans, S2 (1.3), S3 & S4 (0.5-1.6)

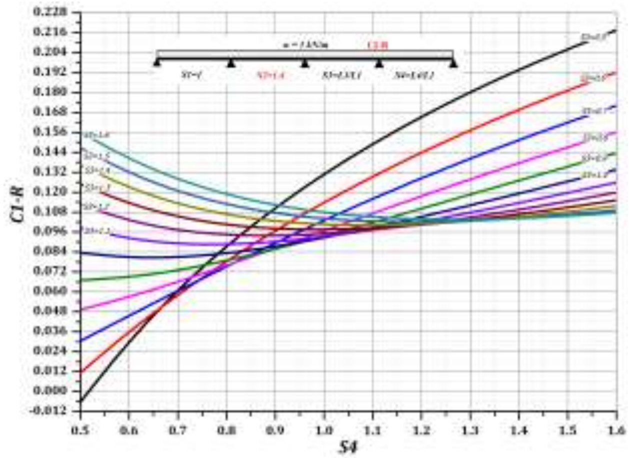


Chart 44: Coefficient C1-R for 4-spans, S2 (1.4), S3 & S4 (0.5-1.6)

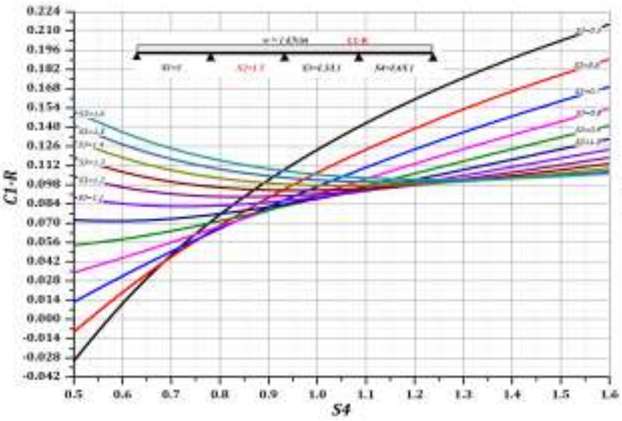


Chart 45: Coefficient C1-R for 4-spans, S2 (1.5), S3 & S4 (0.5-1.6)

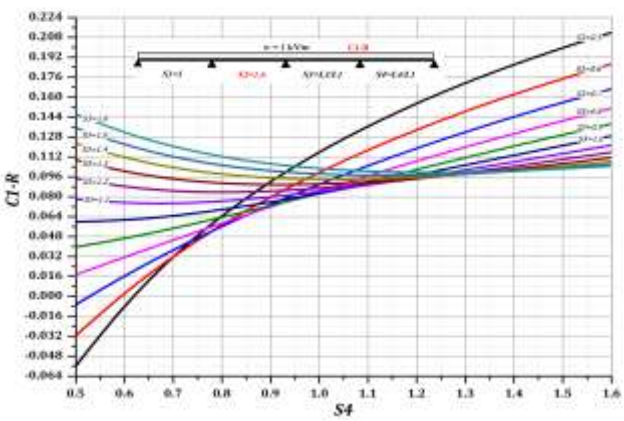


Chart 46: Coefficient C1-R for 4-spans, S2 (1.6), S3 & S4 (0.5-1.6)

Charts for Positive Moments Coefficients for the continuous beams of 4-Spans :

In the following is the charts to extract the negative moments coefficient (D1-L, D2-L, D2-R and D1-R) for the different ratios of S2, S3 and S4 range between 0.5 and 1.6.

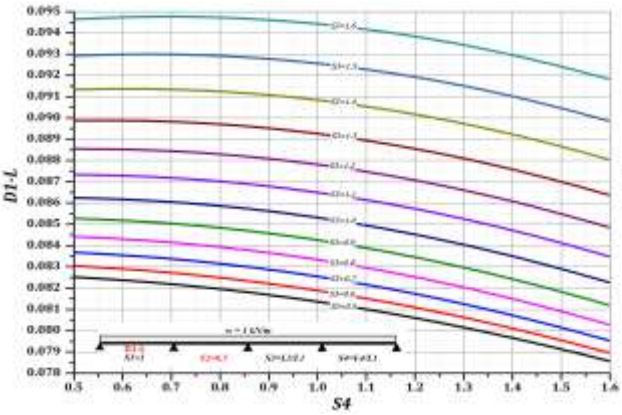


Chart 47: Coefficient D1-L for 4-spans, S2 (0.5), S3 & S4 (0.5-1.6)

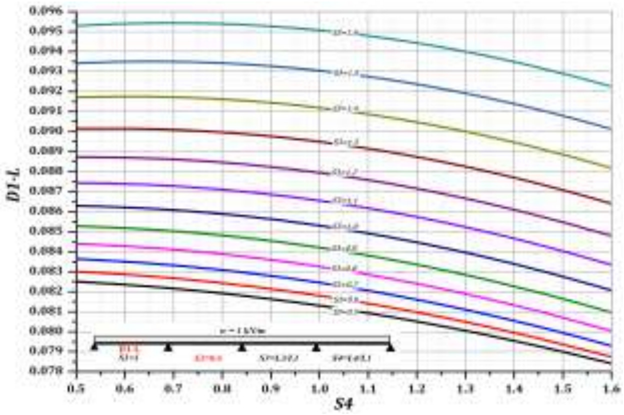


Chart 48: Coefficient D1-L for 4-spans, S2 (0.6), S3 & S4 (0.5-1.6)

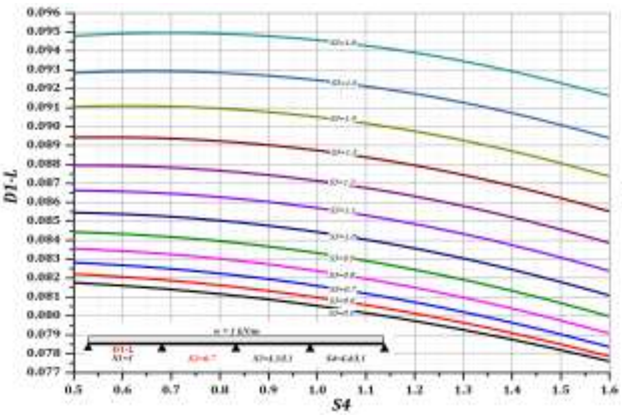


Chart 49: Coefficient D1-L for 4-spans, S2 (0.7), S3 & S4 (0.5-1.6)

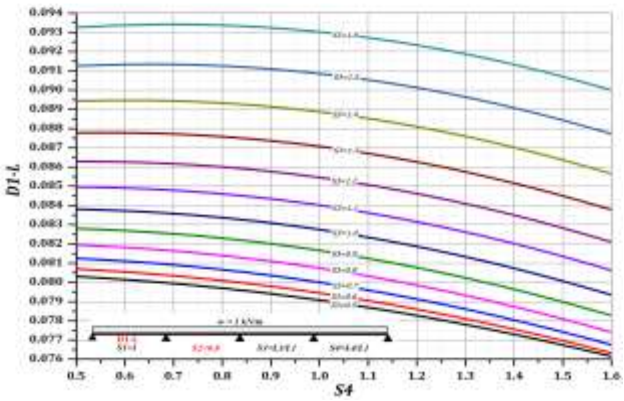
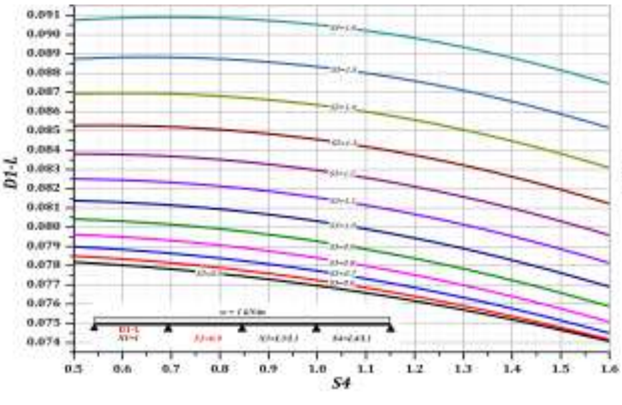


Chart 50: Coefficient D1-L for 4-spans, S2 (0.8), S3 & S4 (0.5-1.6)



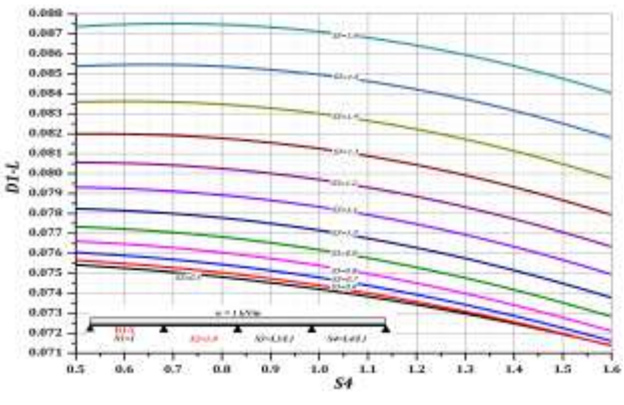
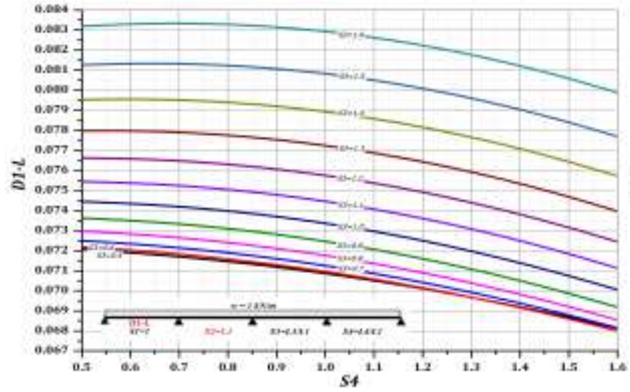


Chart 51: Coefficient D1-L for 4-spans, S2 (0.9), S3 & S4 (0.5-1.6) **Chart 52**: Coefficient D1-L for 4-spans, S2 (1.0), S3 & S4 (0.5-1.6)



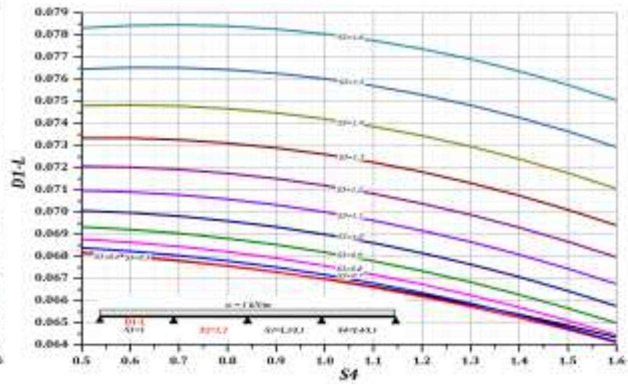
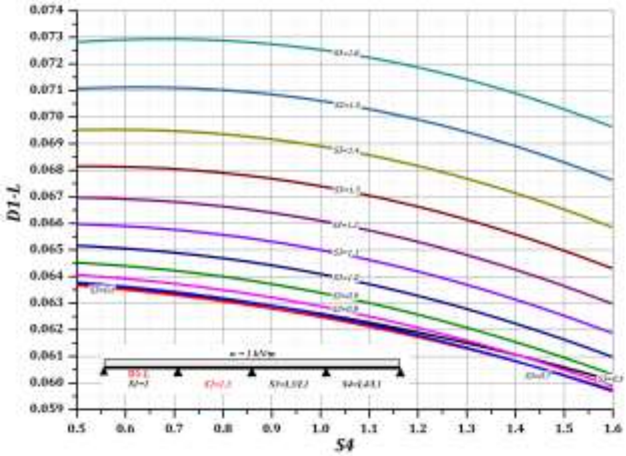


Chart 53: Coefficient D1-L for 4-spans, S2 (1.1), S3 & S4 (0.5-1.6) **Chart 54**: Coefficient D1-L for 4-spans, S2 (1.2), S3 & S4 (0.5-1.6)



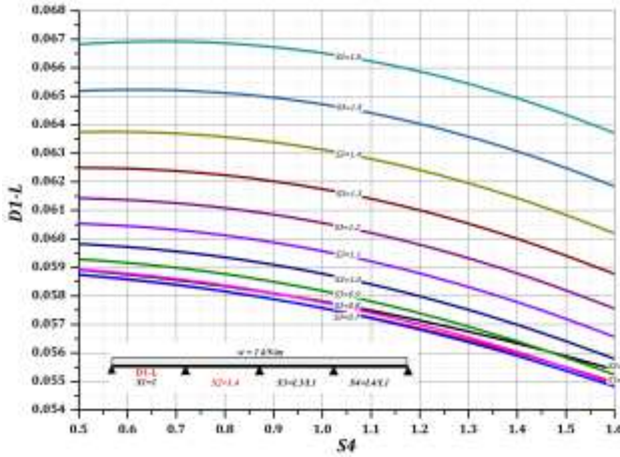
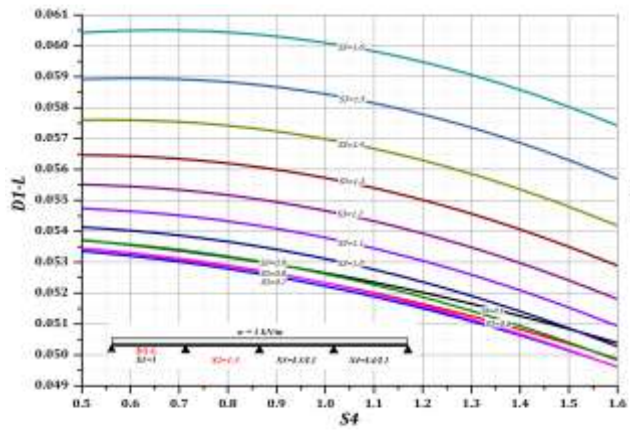


Chart 55: Coefficient D1-L for 4-spans, S2 (1.3), S3 & S4 (0.5-1.6) **Chart 56**: Coefficient D1-L for 4-spans, S2 (1.4), S3 & S4 (0.5-1.6)



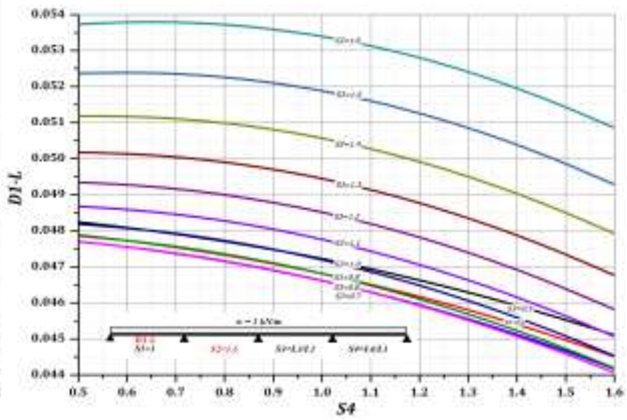
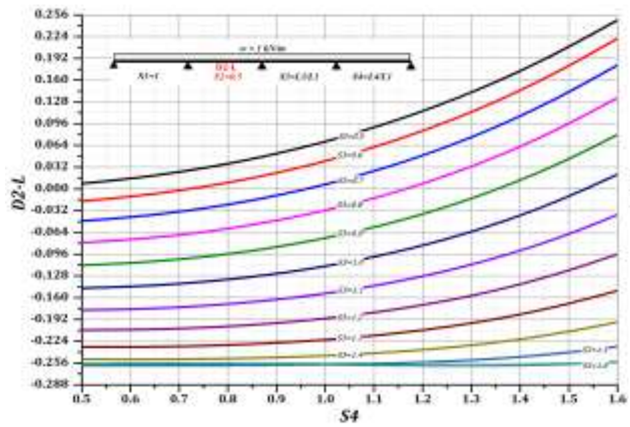


Chart 57: Coefficient D1-L for 4-spans, S2 (1.5), S3 & S4 (0.5-1.6) **Chart 58**: Coefficient D1-L for 4-spans, S2 (1.6), S3 & S4 (0.5-1.6)



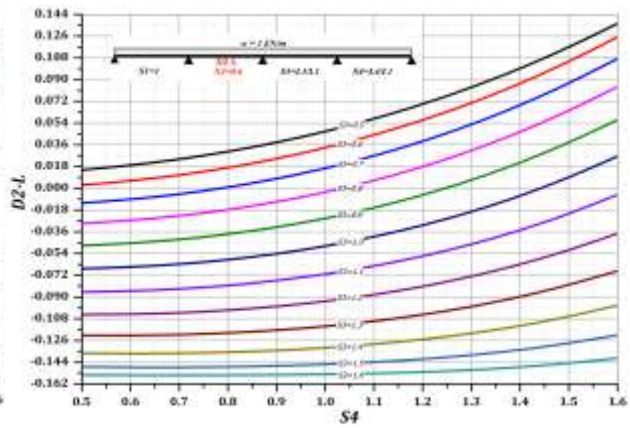


Chart 59: Coefficient D2-L for 4-spans, S2 (0.5), S3 & S4 (0.5-1.6) **Chart 60**: Coefficient D2-L for 4-spans, S2 (0.6), S3 & S4 (0.5-1.6)

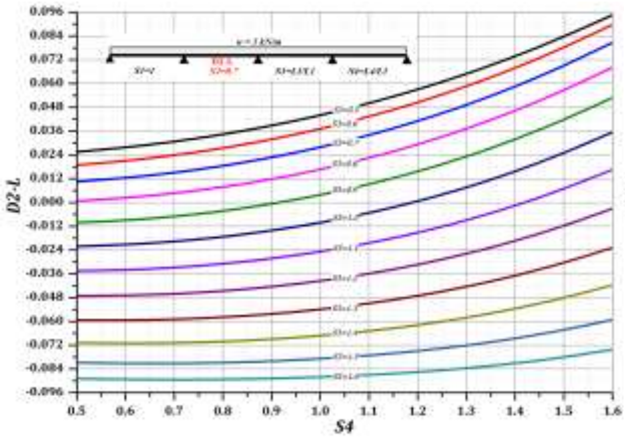


Chart 61: Coefficient D2-L for 4-spans, S2 (0.7), S3 & S4 (0.5-1.6)

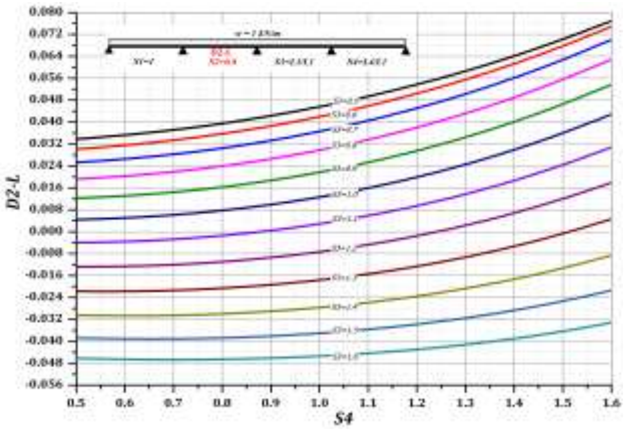


Chart 62: Coefficient D2-L for 4-spans, S2 (0.8), S3 & S4 (0.5-1.6)

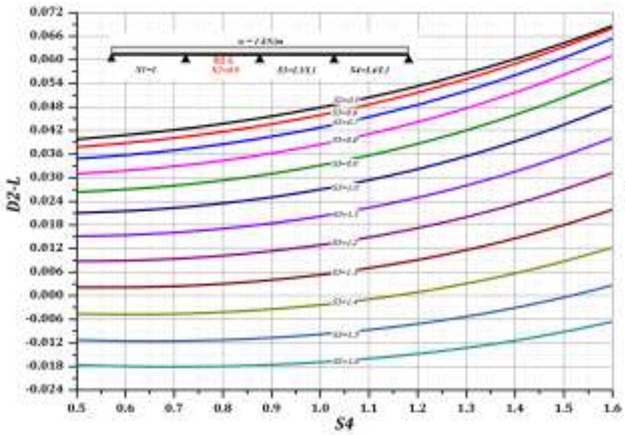


Chart 63: Coefficient D2-L for 4-spans, S2 (0.9), S3 & S4 (0.5-1.6)

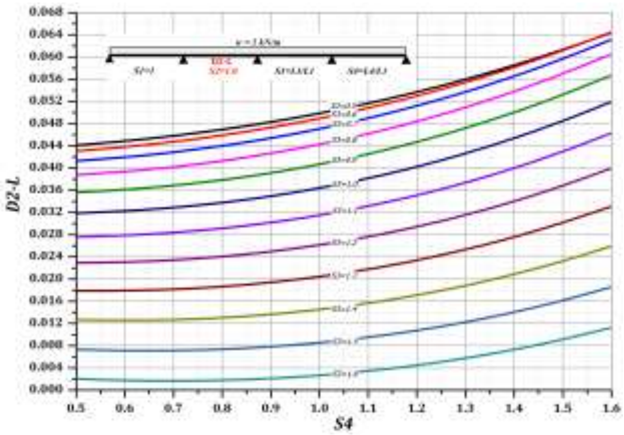


Chart 64: Coefficient D2-L for 4-spans, S2 (1.0), S3 & S4 (0.5-1.6)

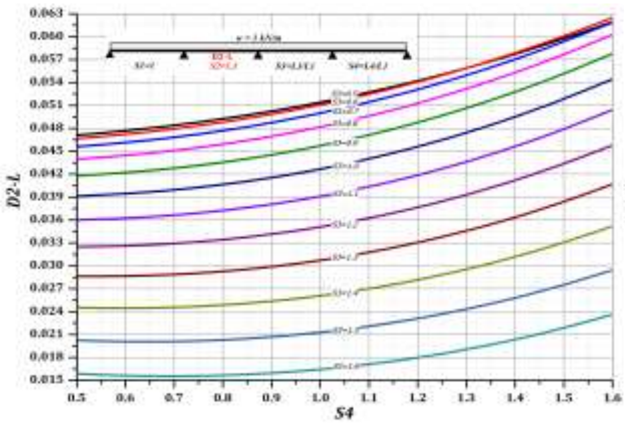


Chart 65: Coefficient D2-L for 4-spans, S2 (1.1), S3 & S4 (0.5-1.6)

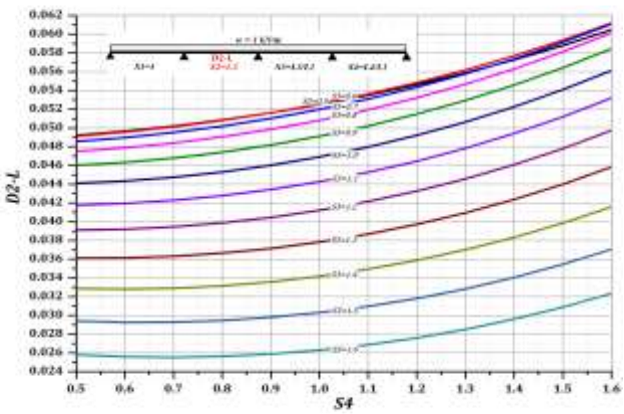


Chart 66: Coefficient D2-L for 4-spans, S2 (1.2), S3 & S4 (0.5-1.6)

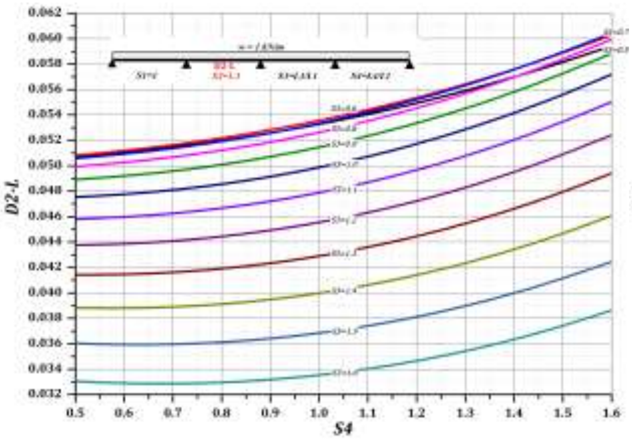


Chart 67: Coefficient D2-L for 4-spans, S2 (1.3), S3 & S4 (0.5-1.6)

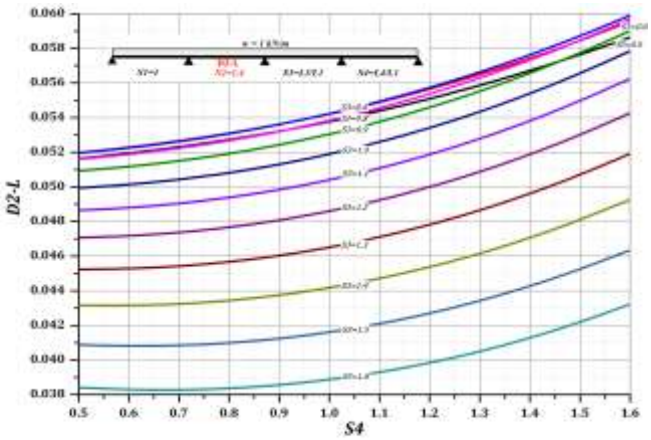


Chart 68: Coefficient D2-L for 4-spans, S2 (1.4), S3 & S4 (0.5-1.6)

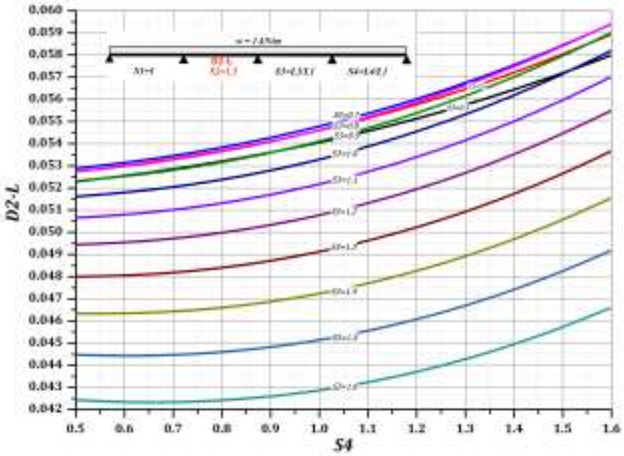


Chart 69: Coefficient D2-L for 4-spans, S2 (1.5), S3 & S4 (0.5-1.6)

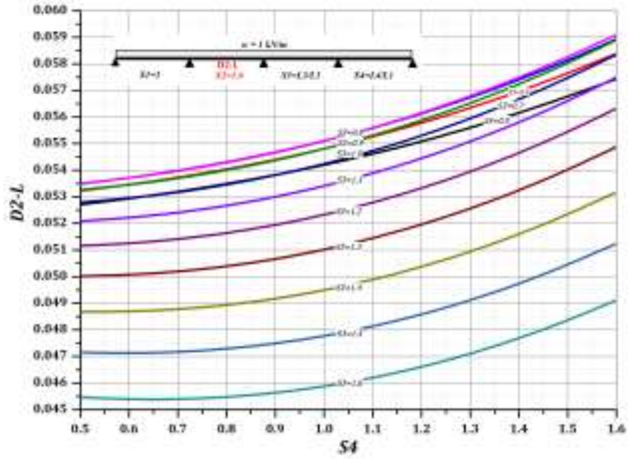


Chart 70: Coefficient D2-L for 4-spans, S2 (1.6), S3 & S4 (0.5-1.6)

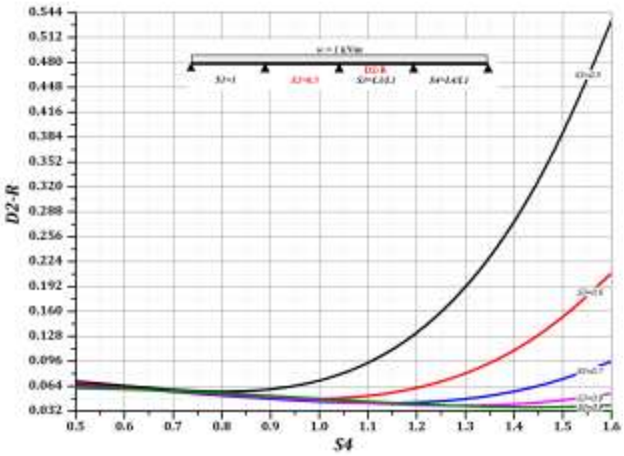


Chart 71: Coefficient D2-R for 4-spans, S2 (0.5), S3 (0.5-0.9) & S4 (0.5-1.6)

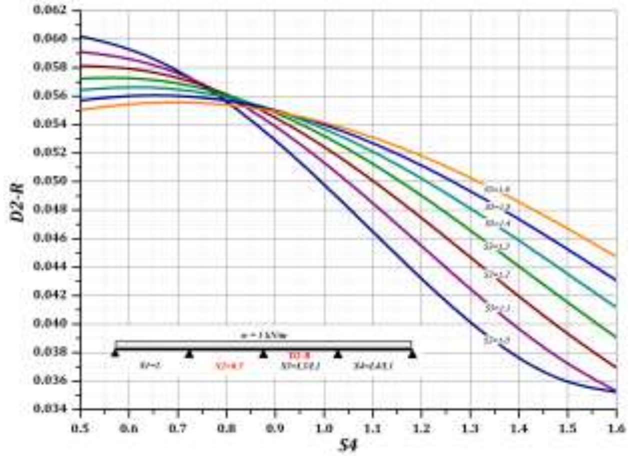


Chart 72: Coefficient D2-R for 4-spans, S2 (0.5), S3 (1.0-1.6) & S4 (0.5-1.6)

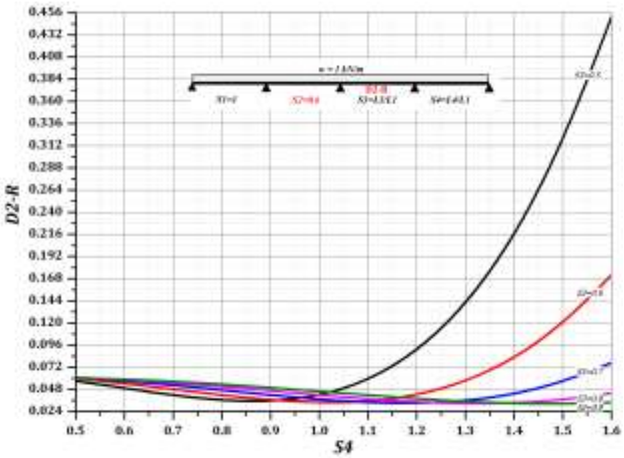


Chart 73: Coefficient D2-R for 4-spans, S2 (0.6), S3 (0.5-0.9) & S4 (0.5-1.6)

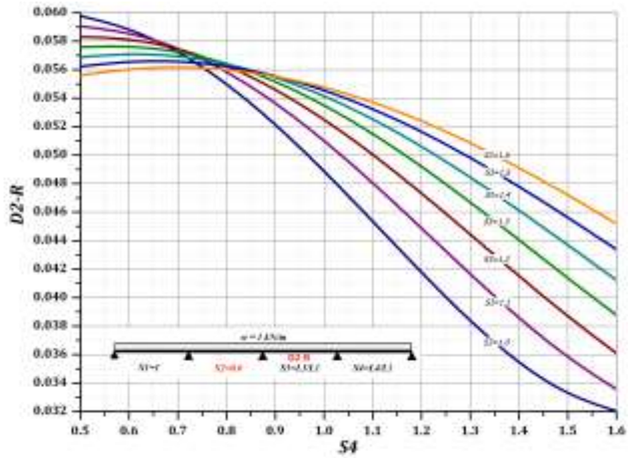


Chart 74: Coefficient D2-R for 4-spans, S2 (0.6), S3 (1.0-1.6) & S4 (0.5-1.6)

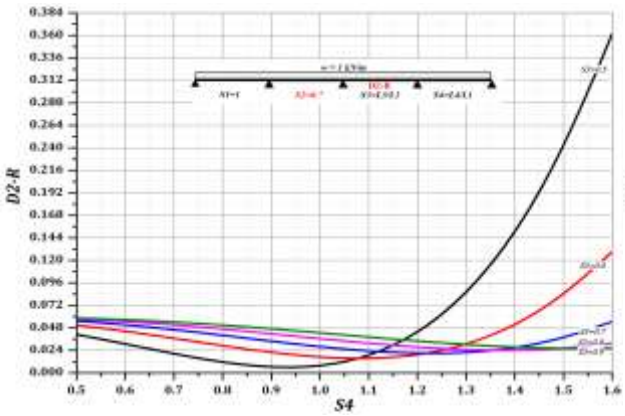


Chart 75: Coefficient D2-R for 4-spans, S2 (0.7), S3 (0.5-0.9) & S4 (0.5-1.6)

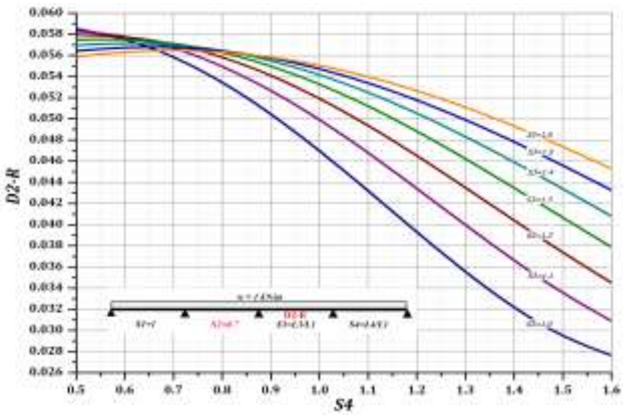


Chart 76: Coefficient D2-R for 4-spans, S2 (0.7), S3 (1.0-1.6) & S4 (0.5-1.6)

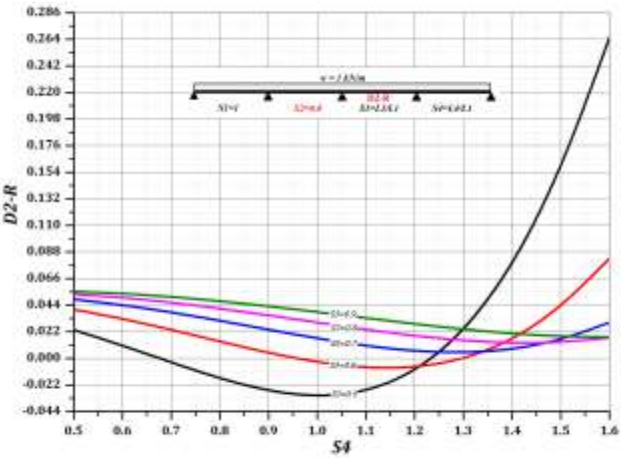


Chart 77: Coefficient D2-R for 4-spans, S2 (0.8), S3 (0.5-0.9) & S4 (0.5-1.6)

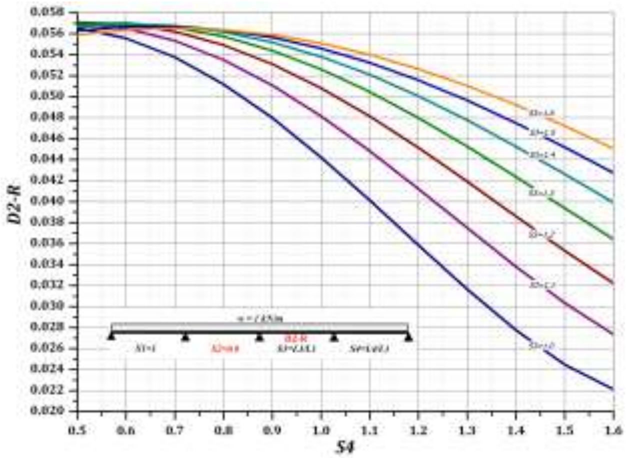


Chart 78: Coefficient D2-R for 4-spans, S2 (0.8), S3 (1.0-1.6) & S4 (0.5-1.6)

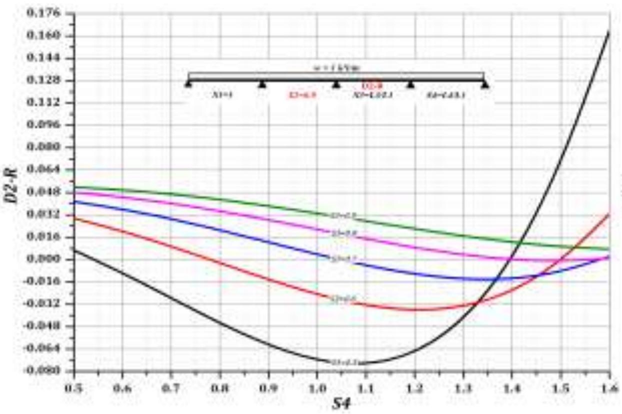


Chart 79: Coefficient D2-R for 4-spans, S2 (0.9), S3 (0.5-0.9) & S4 (0.5-1.6)

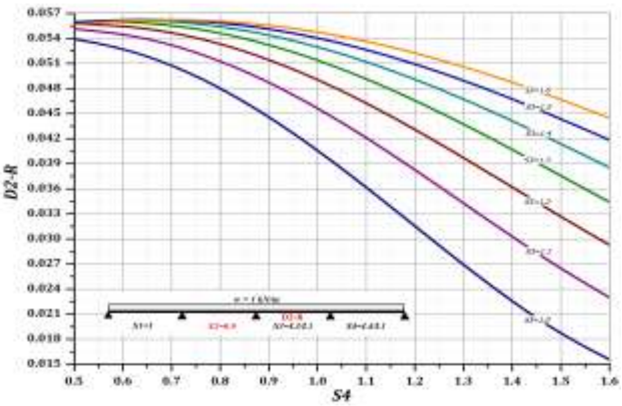


Chart 80: Coefficient D2-R for 4-spans, S2 (0.9), S3 (1.0-1.6) & S4 (0.5-1.6)

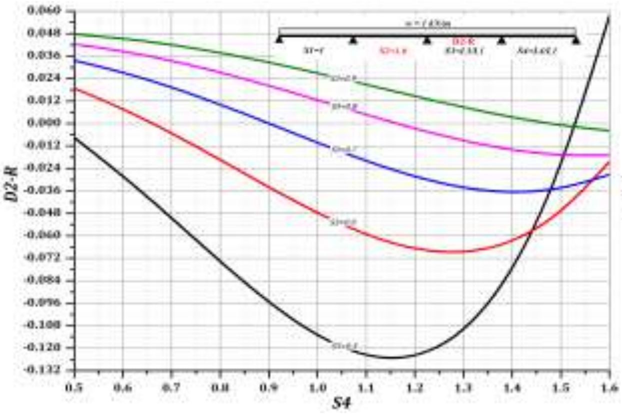


Chart 81: Coefficient D2-R for 4-spans, S2 (1.0), S3 (0.5-0.9) & S4 (0.5-1.6)

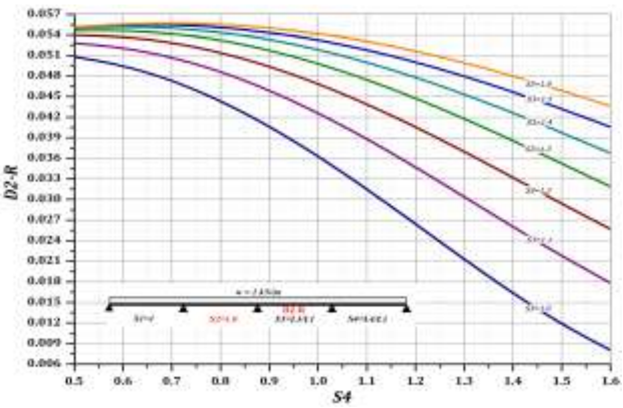


Chart 82: Coefficient D2-R for 4-spans, S2 (1.0), S3 (1.0-1.6) & S4 (0.5-1.6)

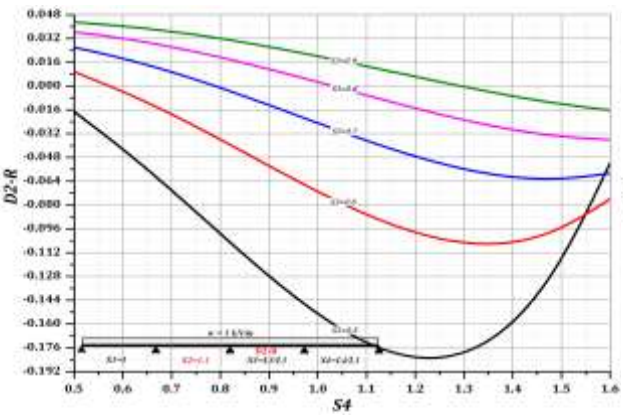


Chart 83: Coefficient D2-R for 4-spans, S2 (1.1), S3 (0.5-0.9) & S4 (0.5-1.6)

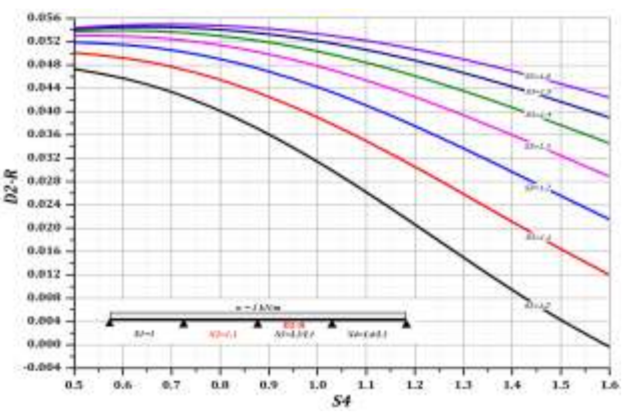


Chart 84: Coefficient D2-R for 4-spans, S2 (1.1), S3 (1.0-1.6) & S4 (0.5-1.6)

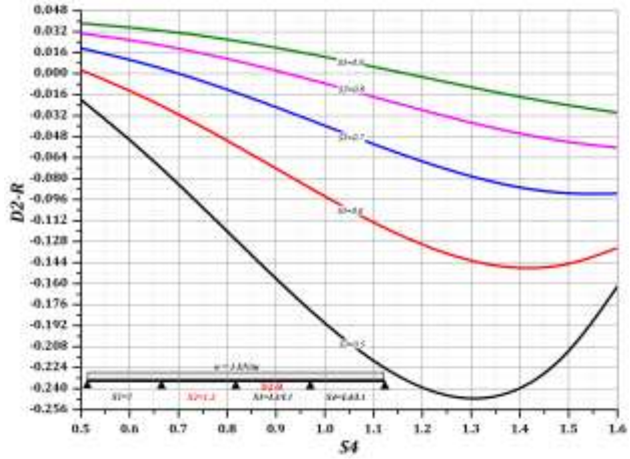


Chart 85: Coefficient D2-R for 4-spans, S2 (1.2), S3 (0.5-0.9) & S4 (0.5-1.6)

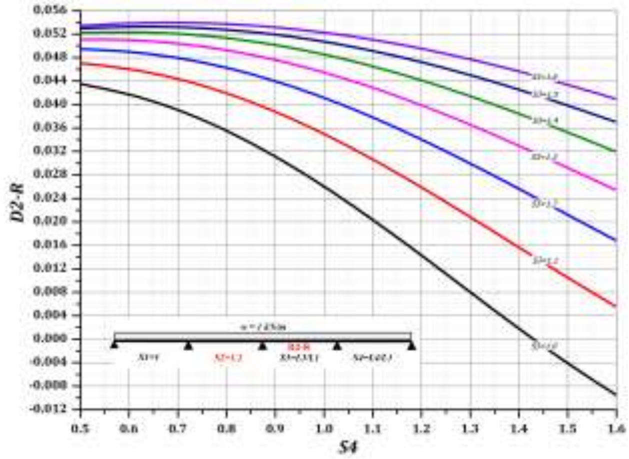


Chart 86: Coefficient D2-R for 4-spans, S2 (1.2), S3 (1.0-1.6) & S4 (0.5-1.6)

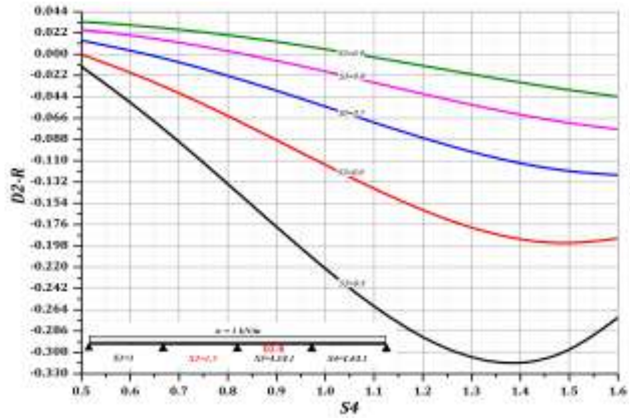


Chart 87: Coefficient D2-R for 4-spans, S2 (1.3), S3 (0.5-0.9) & S4 (0.5-1.6)

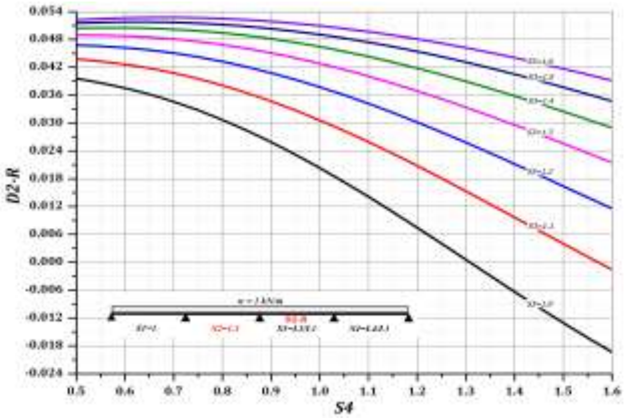


Chart 88: Coefficient D2-R for 4-spans, S2 (1.3), S3 (1.0-1.6) & S4 (0.5-1.6)

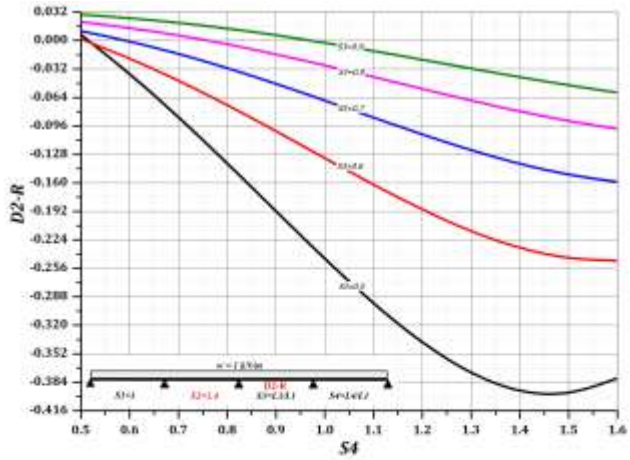


Chart 89: Coefficient D2-R for 4-spans, S2 (1.4), S3 (0.5-0.9) & S4 (0.5-1.6)

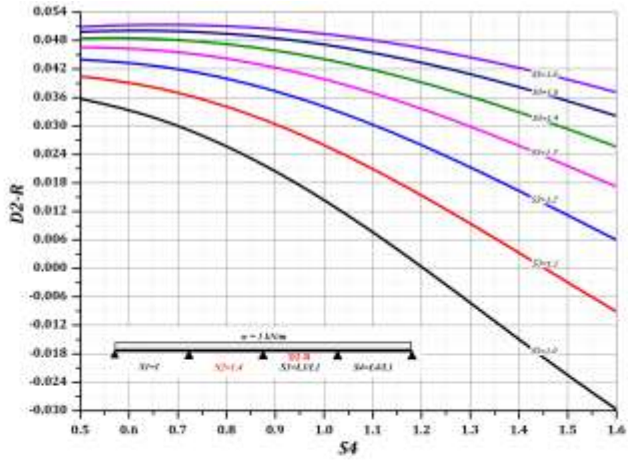


Chart 90: Coefficient D2-R for 4-spans, S2 (1.4), S3 (1.0-1.6) & S4 (0.5-1.6)

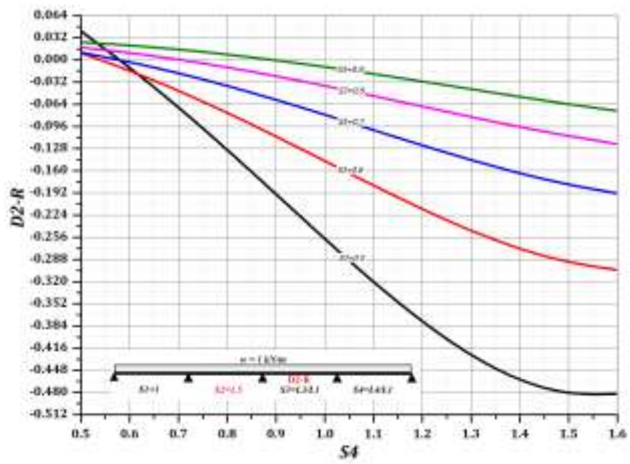


Chart 91: Coefficient D2-R for 4-spans, S2 (1.5), S3 (0.5-0.9) & S4 (0.5-1.6)

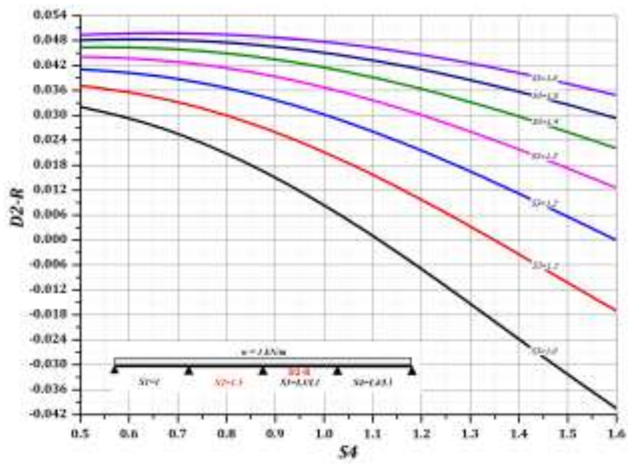


Chart 92: Coefficient D2-R for 4-spans, S2 (1.5), S3 (1.0-1.6) & S4 (0.5-1.6)

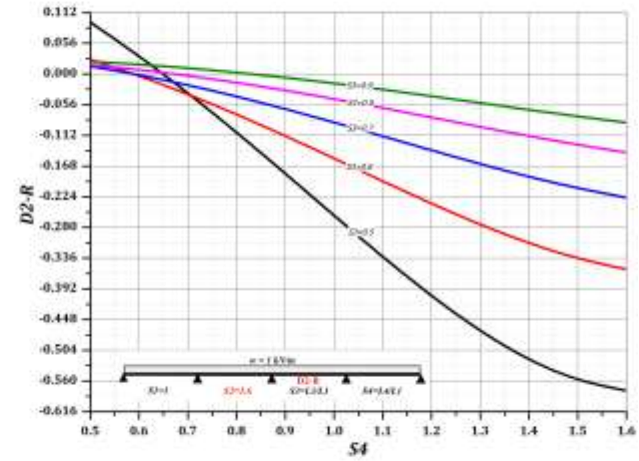


Chart 93: Coefficient D2-R for 4-spans, S2 (1.6), S3 (0.5-0.9) & S4 (0.5-1.6)

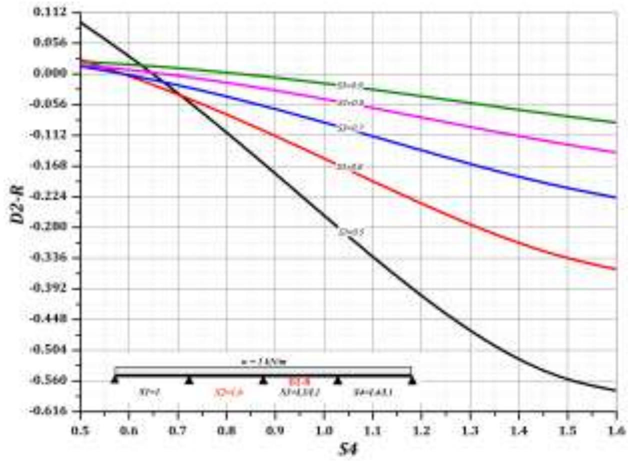


Chart 94: Coefficient D2-R for 4-spans, S2 (1.6), S3 (1.0-1.6) & S4 (0.5-1.6)

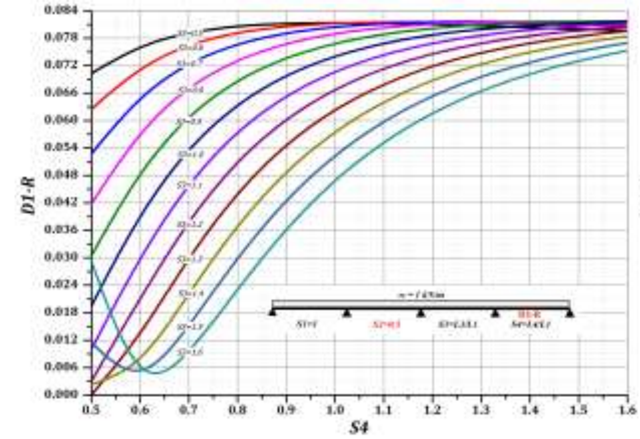


Chart 95: Coefficient D1-R for 4-spans, S2 (0.5), S3 & S4 (0.5-1.6)

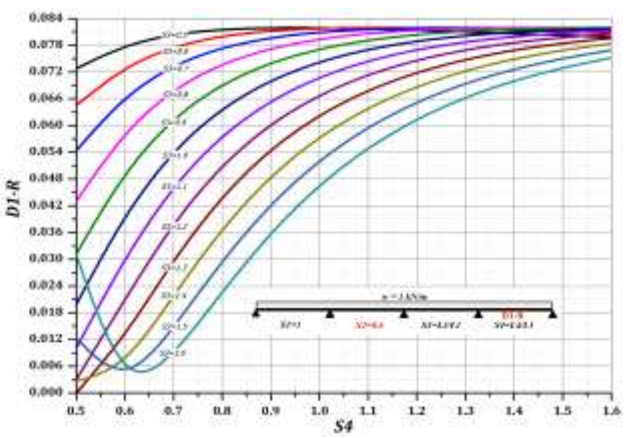


Chart 96: Coefficient D1-R for 4-spans, S2 (0.6), S3 & S4 (0.5-1.6)

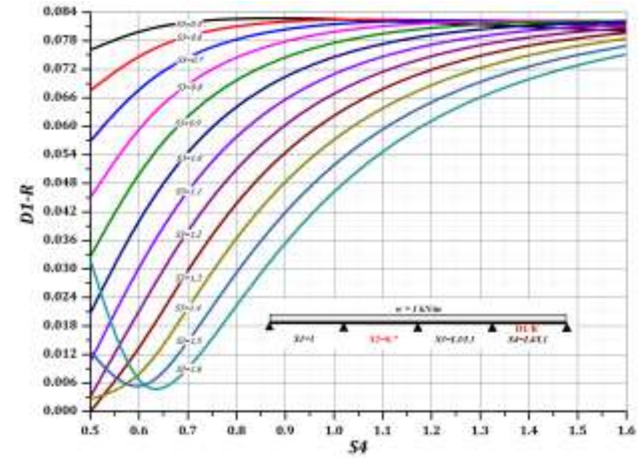


Chart 97: Coefficient D1-R for 4-spans, S2 (0.7), S3 & S4 (0.5-1.6)

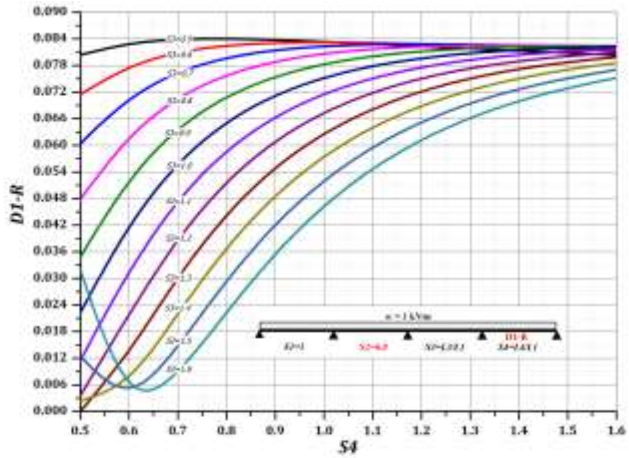


Chart 98: Coefficient D1-R for 4-spans, S2 (0.8), S3 & S4 (0.5-1.6)

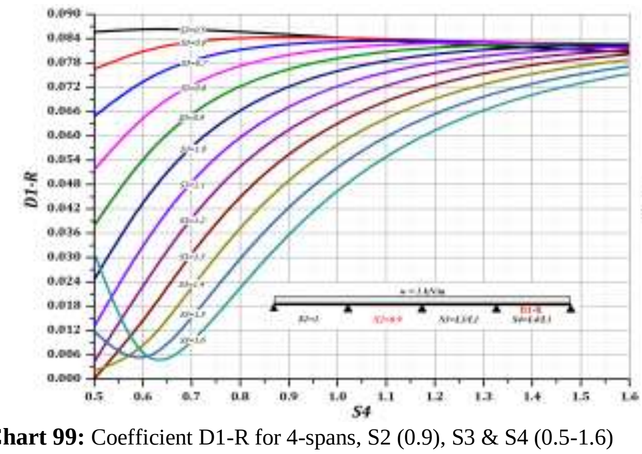


Chart 99: Coefficient D1-R for 4-spans, S2 (0.9), S3 & S4 (0.5-1.6)

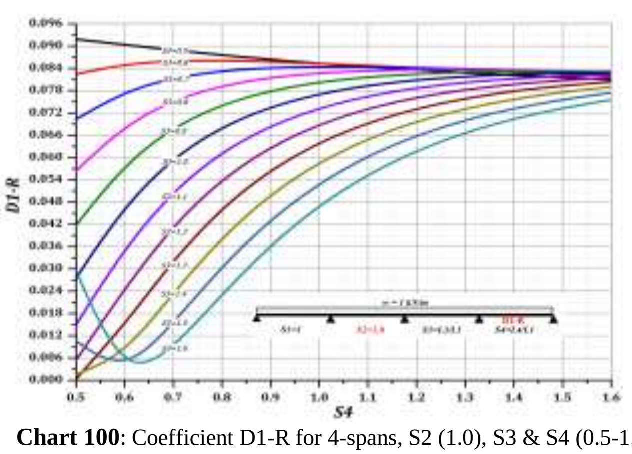


Chart 100: Coefficient D1-R for 4-spans, S2 (1.0), S3 & S4 (0.5-1.6)

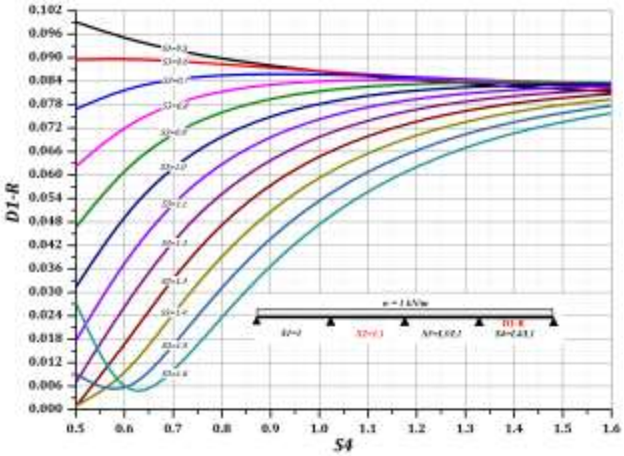


Chart 101: Coefficient D1-R for 4-spans, S2 (1.1), S3 & S4 (0.5-1.6)

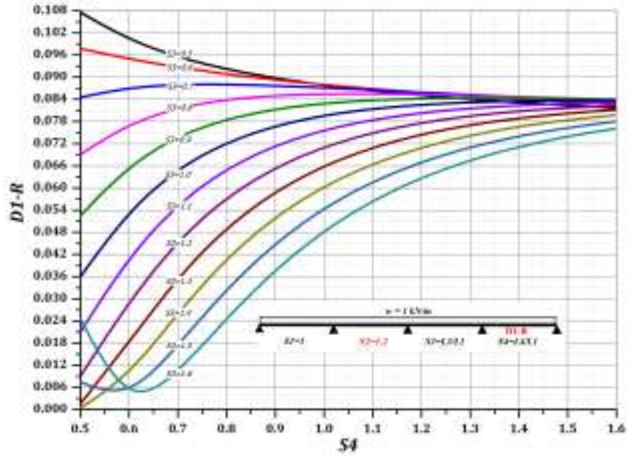


Chart 102: Coefficient D1-R for 4-spans, S2 (1.2), S3 & S4 (0.5-1.6)

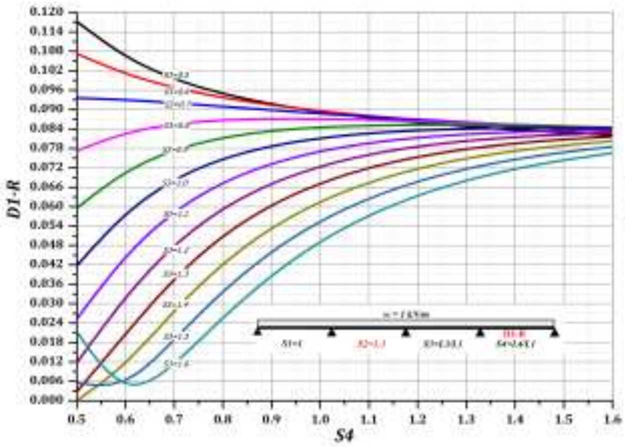


Chart 103: Coefficient D1-R for 4-spans, S2 (1.3), S3 & S4 (0.5-1.6)

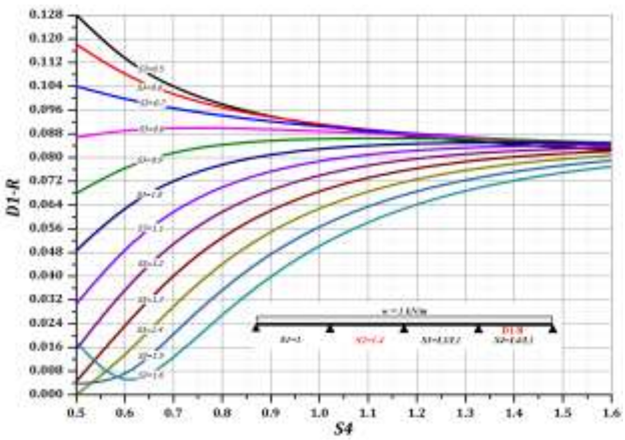


Chart 104: Coefficient D1-R for 4-spans, S2 (1.4), S3 & S4 (0.5-1.6)

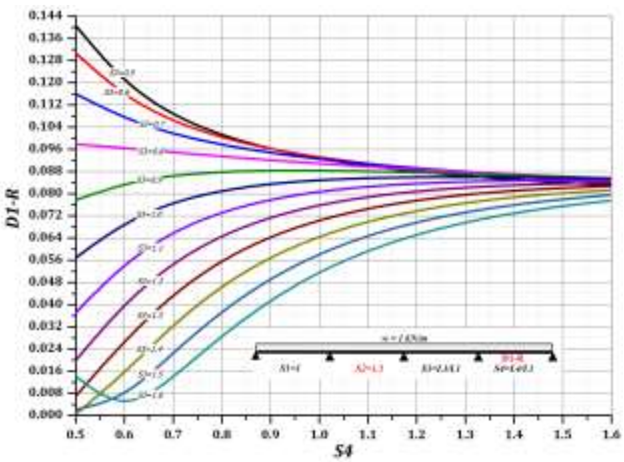


Chart 105: Coefficient D1-R for 4-spans, S2 (1.5), S3 & S4 (0.5-1.6)

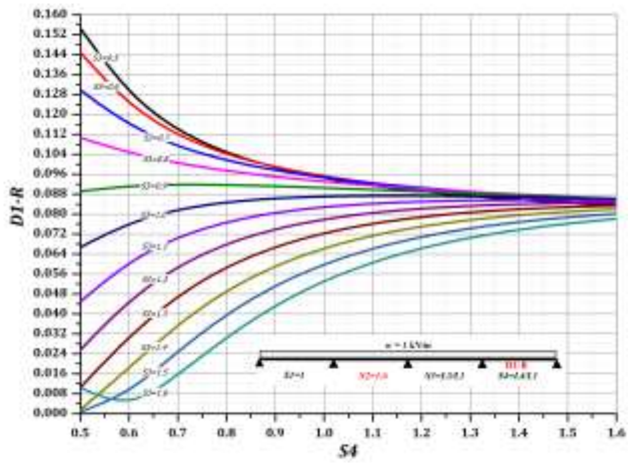


Chart 106: Coefficient D1-R for 4-spans, S2 (1.6), S3 & S4 (0.5-1.6)

VI. CALCULATION OF MOMENTS USING MOMENTS COEFFICIENTS:

In order to calculate negative moment after extracting the moment coefficients from charts and where the coefficients is depend on the unit value of the first span, unit value of uniformly distributed load and corresponding span length, on this the moment can be calculated by using the following steps:

1. For the given span lengths find the ratio of spans based on the first span by dividing each span by L1.
2. For the given number of spans and S calculated, select the matching Charts and extract coefficients.
3. For the given value of uniformly distributed load calculate the negative moment from Equation (5).

$$M = C\, w\, L_{av}^2 \qquad (5)$$

Where:

C is negative moment coefficients extract from charts.

w is the given value of uniformly distributed load.

$L_{av} = ½(L_L + L_R)$ *, where L_L and L_R is the span lengths to the left and to the right to support coincides to C.*

and calculate the positive moment from Equation (6).

$$M = D\ w\ L^2 \qquad (6)$$

Where:

D is positive moment coefficients extract from charts.
L is the span length coincides to D.

VII. NUMERICAL EXAMPLES:

In the following is the some examples to examine the accuracy of moments calculated using coefficients from charts with the moment obtained by using PROKON Software.

Example 1: Continuous Beam of 2-spans.

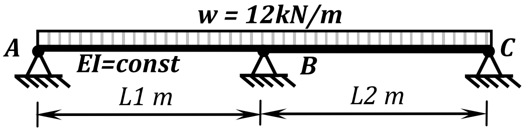


Figure 9: Continuous Beam of 2-Spans

Different span lengths were used and the ratios are calculated accordingly. The moment required to be calculated is the moment at span AB, joint B and span BC and their relative coefficient are D1-L, C1 and D1-R. The input data and calculations are shown in Table(1).

Table (1): Calculations of Moments for Example 1

Ex. No.		Span AB	Joint B	Span BC
1-1	L(m)	4.0		4.0
	S	1		1
	Coefficients	D1-L (Chart 2)	C1 (Chart 1)	D1-R (Chart 2)
		0.070	0.125	0.070
	Moment Type	Sagging	Hogging	Sagging
	M kN.m	13.44	24.0	13.44
	M(PROKON)	13.50	24.0	13.50
	Dif%	0.44%	0.00%	0.44%
1-2	L(m)	4.0		4.2
	S	1		1.05
	Coefficients	D1-L (Chart 2)	C1 (Chart 1)	D1-R (Chart 2)
		0.068	0.126	0.072
	Moment Type	Sagging	Hogging	Sagging
	M kN.m	13.06	25.42	15.24
	M(PROKON)	13.03	25.26	15.34
	Dif%	0.23%	0.63%	0.65%
1-3	L(m)	4.8		4.0
	S	1		0.83
	Coefficients	D1-L (Chart 2)	C1 (Chart 1)	D1-R (Chart 2)
		0.077	0.129	0.058
	Moment Type	Sagging	Hogging	Sagging
	M kN.m	21.29	29.97	11.14
	M(PROKON)	21.28	29.76	11.43
	Dif%	0.05%	0.71%	2.54%
1-4	L(m)	4		6
	S	1		1.5
	Coefficients	D1-L (Chart 2)	C1 (Chart 1)	D1-R (Chart 2)
		0.040	0.140	0.081
	Moment Type	Sagging	Hogging	Sagging
	M kN.m	7.68	42.00	34.99
	M(PROKON)	7.59	42.00	35.04
	Dif%	1.19%	0.00%	0.14%

Example 2: Continuous Beam of 3-spans.

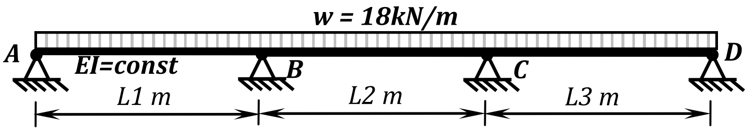


Figure 9: Continuous Beam of 3-Spans

Different span lengths were used and the ratios are calculated accordingly. The moment required to be calculated is the moment at span AB, joint B, span BC, joint C and span CD and their relative coefficient are D1-L, C1-L, C1-R and D1-R. The input data and calculations are shown in Table(2)

Table (2): Calculations of Moments for Example 2

Ex. No.		Span AB	Joint B	Span BC	Joint C	Span CD

Ex. No.		Span AB	Joint B	Span BC	Joint C	Span CD
2-1	L(m)	4.0		4.0		4.0
	S	S1=1		S2=1		S3=1
	Coefficients	D1-L (Chart 5)	C1-L (Chart 3)	D2 (Chart 7)	C1-R (Chart 4)	D1-R (Chart 10)
		0.081	0.100	0.025	0.100	0.081
	Moment Type	Sagging	Hogging	Sagging	Hogging	Sagging
	M kN.m	23.33	28.80	7.20	28.80	23.33
	M(PROKON)	23.04	28.80	7.20	28.80	23.04
	Dif%	1.26%	0.00%	0.00%	0.00%	1.26%
2-2	L(m)	4.0		6.0		8.0
	S	1		1.5		2.0
	Coefficients	D1-L (Chart 5)	C1-L (Chart 3)	D2 (Chart 7)	C1-R (Chart 4)	D1-R (Chart 10)
		0.079	0.066	0.024	0.126	0.082
	Moment Type	Sagging	Hogging	Sagging	Hogging	Sagging
	M kN.m	22.75	29.70	15.55	111.13	94.46
	M(PROKON)	22.64	29.82	15.82	110.61	94.00
	Dif%	0.52%	0.39%	1.71%	0.47%	0.49%
2-3	L(m)	8.0		4.0		6.0
	S	1		0.50		0.75
	Coefficients	D1-L (Chart 5)	C1-L (Chart 3)	D2 (Chart 6)	C1-R (Chart 4)	D1-R (Chart 8)
		0.084	0.154	-0.104	0.094	0.094
	Moment Type	Sagging	Hogging	Hogging	Hogging	Sagging
	M kN.m	96.77	99.79	29.95	42.30	60.91
	M(PROKON)	97.98	100.86	30.00	42.83	61.00
	Dif%	1.24%	1.06%	0.15%	1.23%	0.15%

Example 3: Continuous Beam of 4-spans.

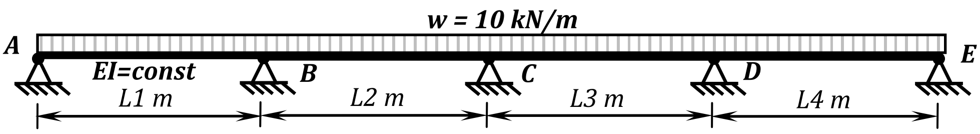


Figure 10: Continuous Beam of 4-Spans

Different span lengths were used and the ratios are calculated accordingly. The moment required to be calculated is the moment at span AB, joint B, span BC, joint C, span CD, joint D and span DE and their relative coefficient are D1-L, C1-L, D2-L, C2, D2-R, C1-R and D1-R. The input data and calculations are shown in Table(3)

Table (3): Calculations of Moments for Example 3

Ex. No.		Span AB	Joint B	Span BC	Joint C	Span CD	Joint D	Span DE
2-1	L(m)	4.0		4.0		4.0		4.0
	S	S1=1		S2=1		S3=1		S4=1
	Coefficients	D1-L (Chart 52)	C1-L (Chart 16)	D2-L (Chart 64)	C2 (Chart 28)	D2-R (Chart 82)	C1-R (Chart 40)	D1-R (Chart 100)
		0.077	0.107	0.036	0.071	0.036	0.107	0.077
	Moment Type	Sagging	Hogging	Sagging	Hogging	Sagging	Hogging	Sagging
	M kN.m	12.32	17.12	5.76	11.36	5.76	17.12	12.32
	M(PROKON)	12.35	17.14	5.82	11.43	5.82	17.14	12.35
	Dif%	0.22%	0.13%	0.97%	0.60%	0.97%	0.13%	0.22%
2-2	L(m)	5.0		8.0		4.0		6.0
	S	1		1.6		0.80		1.2
	Coefficients	D1-L (Chart 58)	C1-L (Chart 22)	D2-L (Chart 70)	C2 (Chart 34)	D2-R (Chart 93)	C1-R (Chart 46)	D1-R (Chart 106)
		0.046	0.117	0.056	0.109	-0.081	0.109	0.091
	Moment Type	Sagging	Hogging	Sagging	Hogging	Hogging	Hogging	Sagging
	M kN.m	11.50	49.43	35.84	39.24	12.96	27.25	32.76
	M(PROKON)	11.48	49.23	35.94	39.06	12.68	27.19	32.43
	Dif%	0.16%	0.41%	0.27%	0.47%	2.18%	0.23%	1.01%

VIII. CONCLUSION:

An elastic analysis was carried out to calculate the negative and positive moment coefficients for the continuous beams of 2-spans, 3-spans and 4-spans with different span lengths controlled by the spans ratios, the spans support uniformly distributed load full span lengths. The analysis has been done according to the moment distribution methods procedure with using spreadsheets for calculations to analyze 2000 models of continuous beams. The negative and positive moments obtained were converted to moment coefficients because of its more generality. The moment coefficients have been derived depending on the span ratios based on the first span. The ratios adopted are ranged between 0.5 and 2.0 for the 2-spans and 3-spans and between 0.5 and 1.6 for the 4-spans. A unit value was taken for the uniformly distributed load The different values of moment coefficients related to spans ratios were explained graphically by a number of 106 charts. For the given case of continuous beam and after calculating the spans ratios, the moment coefficients can be very easily extracted from the matching charts. The values of moment can be calculated using the appropriate formula according to the coefficients extracted from charts.

The values of coefficients were verified by selecting some examples of continuous beams consisting of 2-span, 3-spans and 4-spans with different span length and the values of moments calculated using the coefficients were appear good agreement with the ones obtained by using PROKON software with observing that the difference percentage in general is less than 1% and the bigger one is 2.18%.

This lead to conclude that the charts derived in this study to calculate the negative and positive moment can be used simply and efficiently in the absence of computer software and without doing complicated calculations.

More study on the behavior of continuous with the variation of span length and analysis of continuous under influence of uniformly distributed load on spans with different values can be recommended for future work.

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